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A SIMPLE ANALYTICAL MODEL
FOR ASYNCHRONOUS DENSE WDM/OOK SYSTEMS

by

Yun-Yao Huang
LTCOL, Taiwan Air Force
B.S, Chung-Cheng Institute of Technology 1978

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ABSTRACT

We derive the closed form expression for the bit error probability of asynchronous dense WDM systems employing an external OOK modulator. Our model is based upon a close approximation of the optical Fabry-Perot filter in the receiver as a single-pole RC filter for signals that are bandlimited to a frequency band approximately equal to one sixtieth of the Fabry-Perot filter's free spectral range. Our model can handle bit rates up to 2.5 Gb/s for a free spectral range of 3800 GHz and up to 5 Gb/s when the power penalty is 1 dB or less.

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I. INTRODUCTION

Asynchronous wavelength division multiplexing (WDM) systems have been increasingly proposed as an attractive alternative to coherent optical frequency division multiplexing (FDM) systems [Ref. 1-5]. Although asynchronous WDM systems with direct detection do not have the channel capacity of coherent optical FDM systems, they are much less costly to implement. Furthermore, present filter technology enables the designers to tightly pack the channel, resulting in asynchronous dense WDM systems that can provide aggregate bit rates of many terabits per second ($1 \text{ Tb/s} = 10^{12} \text{ b/s}$). Asynchronous dense WDM systems are particularly attractive in the area of undersea surveillance where hundreds of sensors and data collection sites are envisioned being merged onto single-fiber superhighways through massive data fusion. Other applications call for relatively low-bandwidth data collection over many months to be dumped quickly to a remote recording site in a matter of minutes. This "collection-and-dumped" compression can demand total data rates on the order of hundreds of Gb/s. Long distance between data collection sites and a remote recording site requires the use of optical amplifiers. Therefore, it is necessary to pack all channels within the optical amplifier bandwidth.

An asynchronous dense WDM receiver with on-off-keying (OOK) modulation can be modeled as shown in Fig. 1. Conceptually, the analysis involves two main operations:

1) a convolution operation to evaluate the signal at the output of the optical filter, a Fabry-Perot (FP) filter in our investigation, and 2) the integration of the output of the photodetector. Evaluation of bit error probability by the numerical analysis of these two operations has been carried out in [Ref. 6], with a number of approximations made to reduce the computational complexity. In this investigation the FP filter is shown to be well approximated by an RC filter within the frequency range $|f - f_0| < FSR / 20\pi$, where FSR is the free spectral range of the FP filter [Ref. 7] and f_0 is the FP filter center frequency. For example, given $FSR = 3800$ GHz, the approximation works very well for $|f - f_0| < 60.5$ GHz; that is, the effects of adjacent channels within a 121 GHz bandwidth centered at f_0 must be included, while all others can be neglected. This simple model agrees well with [Ref. 6] as demonstrated in Section III. Furthermore, this model enables us to obtain a closed form analytical expression for the bit error probability for which numerical results can be obtained with little effort. Our investigation shows that this simple model provides accurate results as compared to those in [Ref. 6] for bit rates up to 2.5 Gb/s when the effects of two adjacent channels are included with $FSR = 3800$ GHz. Actually, when the power penalty relative to single channel operation is 1 dB or less, there is virtually no difference in the effect of four or two adjacent channels. Thus, for this power penalty criterion, this simple model can handle bit rates up to 5 Gb/s for a FP filter's $FSR = 3800$ GHz.

In Section II the closed form expression for the decision variable, and consequently, the bit error probability assuming all channels are bit asynchronous as in [Ref. 6] is

derived. Section III presents the numerical results which include the bit error probability versus the signal-to-noise ratio as function of the FP filter bandwidth and channel spacing, and the power penalty (relative to single channel operation without filtering or with filtering but no intersymbol interference) versus the channel spacing as a function of the bandwidth. Finally, a summary of results appears in Section IV.

II. ANALYSIS

The receiver model for the asynchronous dense WDM system is shown in Fig. 1. The desired signal is filtered by a Fabry-Perot (FP) filter that rejects adjacent channels. The photodetector is assumed to have a responsivity \mathcal{R} (A/W). The detected current is amplified by a low noise amplifier that contributes a postdetection thermal noise $n(t)$ with spectral density N_o (A²/Hz). The decision variable at the output of the integration is compared to a threshold α to determine whether a bit zero or bit one was present.

A. INPUT SIGNAL

For convenience, we designate channel 0 as the desired channel, and channel k as an adjacent channel where $k = -M/2, \dots, -1, 1, \dots, M/2$ with M an even integer. We consider the equivalent lowpass (complex envelope) data signal in channel 0 and channel k as follows:

$$b_0(t) = \sum_{i=-L_0}^0 b_{0,i} P_T(t - iT) \quad (1)$$

$$b_k(t) = \sum_{l=-L}^0 b_{k,l} e^{j\omega_k(t-\tau_k)} P_T(t - (lT + \tau_k)) \quad (2)$$

where

T : bit duration

$b_{0,i} \in \{0, 1\}$: bit in channel 0 in the time interval $(iT, (i+1)T)$

$b_{k,l} \in \{0, e^{j\phi_k}\}$ is the l^{th} bit in channel k in the time interval $(lT + \tau_k, (l+1)T + \tau_k)$
 ϕ_k : a phase offset between channel k and channel 0 and is assumed to be uniformly
distributed in $(0, 2\pi)$ radians

ω_k : radian frequency spacing between channel k and channel 0 with $\omega_k = -\omega_{-k}$
 τ_k : is the time delay between channel k and channel 0 and is assumed to be
uniformly distributed in $(0, T)$.

The function $P_T(t - iT)$ is defined as

$$P_T(t - iT) = \begin{cases} 1, & iT \leq t < (i+1)T \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

In both (1) and (2), the non-negative integers L_0 and L represent the number of bits in
channel 0 and k , respectively, that proceed the detected bits $b_{0,0}$. The received
asynchronous dense WDM equivalent lowpass signal at the input of the FP filter is given
by

$$r(t) = \sqrt{P} b_0(t) + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \sqrt{P} b_k(t) \quad (4)$$

where P is the received optical power.

B. FABRY-PEROT FILTERED OUTPUT SIGNAL

The FP filter can be characterized by the following equivalent lowpass transfer function [Ref.1,7]

$$\begin{aligned}
 H(f) &= \frac{1-\rho}{1-\rho e^{-j2\pi f/FSR}} \bullet \frac{1-A-\rho}{1-\rho} \\
 &= \frac{1-\rho}{1-\rho \cos(\frac{2\pi f}{FSR})+j\rho \sin(\frac{2\pi f}{FSR})} \bullet \frac{1-A-\rho}{1-\rho} \quad (5)
 \end{aligned}$$

where ρ is the power reflectivity, A is the power absorption loss (zero for an ideal FP filter) and FSR is the free spectral range. For $|f| < FSR/20\pi$ and assuming $A = 0$, we can approximate $H(f)$ as follow:

$$\begin{aligned}
 H(f) &\approx \frac{1-\rho}{(1-\rho)+j\frac{2\pi\rho f}{FSR}} = \frac{1}{1+j\frac{2\pi\rho f}{(1-\rho)FSR}} \\
 &\approx \frac{1}{1+j\frac{2\pi f}{c}} , \quad |f| < FSR/20\pi \quad (6a)
 \end{aligned}$$

where

$$c = \frac{FSR(1-\rho)}{\rho} \quad (6b)$$

The free spectral range FSR can be related to the full width at half maximum (FWHM) bandwidth B and the finesse F of the FP filter as

$$FSR = \frac{\pi\sqrt{\rho} B}{1-\rho} = BF \quad (7)$$

Thus if the signal is bandlimited to $|f| < FSR/20\pi$, we can truly approximate (5) with a single-pole RC filter with the following transfer function and impulse response

$$H(f) = \frac{1}{1+j\frac{2\pi f}{c}} \quad (8)$$

$$h(t) = ce^{-ct}, \quad t > 0 \quad (9)$$

Figures 2a-b show the magnitude and phase (radians) of $H(f)$ of the FP filter in (5) and its single-pole RC filter approximation given in (8) for $\rho = 0.99$, $F = 312.6$, $B = 12.16$ GHz and $FSR = 3800$ GHz. Note that as the frequency increases, the phases of the FP filter and the RC filter differ markedly, but the magnitudes of their transfer functions remain identical and attenuate rapidly. When $|f| > FSR/20\pi$, the magnitude of $H(f)$ is very small, and therefore, the effect of adjacent channel interference beyond this frequency range is negligible. Fig. 3 shows the normalized impulse response of both FP and single-pole RC filters. In summary, the above approximation is valid for asynchronous dense WDM analysis when the filter finesse F is large or equivalently the FWHM bandwidth B is small since the equivalent lowpass signal must be bandlimited to about $|f| < FSR/20\pi$.

This approximation has been used in [Ref. 5] to study spectral efficiency of optical FDM/ASK systems, which involves the evaluation of the decision variable for worst-case

analysis using the eye diagram technique. Since we are interested in the detected bit $b_{0,0}$ in the time interval $(0, T)$, we consider the output filtered signal $s(t)$, $0 < t \leq T$ given by

$$s(t) = s_B(t) + s_{ISI}(t) + s_{ACI}(t), \quad 0 < t \leq T \quad (10)$$

where

$s_B(t)$: desired signal

$s_{ISI}(t)$: intersymbol interference (ISI) signal

$s_{ACI}(t)$: adjacent channel interference (ACI) signal

These signals are evaluated using (4) and (9) as follows (detailed derivations in Appendix A):

$$\begin{aligned} s_B(t) &= \sqrt{P} b_{0,0} \int_0^t h(t-\lambda) d\lambda \\ &= \sqrt{P} b_{0,0} (1 - e^{-ct}), \quad 0 < t \leq T \end{aligned} \quad (11)$$

$$\begin{aligned} s_{ISI}(t) &= \sqrt{P} \sum_{i=-L_0}^{-1} b_{0,i} \int_{iT}^{(i+1)T} h(t-\lambda) d\lambda \\ &= \sqrt{P} e^{-ct} \sum_{i=-L_0}^{-1} b_{0,i} (e^{(i+1)cT} - e^{icT}), \quad 0 < t \leq T \end{aligned} \quad (12)$$

$$S_{ACI}(t)$$

$$\begin{aligned}
&= \sqrt{P} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \left[\sum_{l=-L}^{-2} b_{k,l} \int_{lT+\tau_k}^{(l+1)T+\tau_k} h(t-\lambda) e^{j\omega_k(\lambda-\tau_k)} d\lambda \right. & 0 < t < T \\
&\quad + b_{k,-1} \int_{-T+\tau_k}^t h(t-\lambda) e^{j\omega_k(\lambda-\tau_k)} d\lambda & 0 < t < \tau_k \\
&\quad + b_{k,-1} \int_{-T+\tau_k}^{\tau_k} h(t-\lambda) e^{j\omega_k(\lambda-\tau_k)} d\lambda & \tau_k < t < T \\
&\quad \left. + b_{k,0} \int_{\tau_k}^t h(t-\lambda) e^{j\omega_k(\lambda-\tau_k)} d\lambda \right] & \tau_k < t < T \\
\\
&= \sqrt{P} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \left[\sum_{l=-L}^{-2} \pi(0, T) b_{k,l} e^{-ct} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} e^{(c+j\omega_k)(lT+\tau_k)} (e^{(c+j\omega_k)T} - 1) \right. \\
&\quad + \pi(0, \tau_k) b_{k,-1} e^{-ct} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} (e^{(c+j\omega_k)t} - e^{(c+j\omega_k)(-T+\tau_k)}) \\
&\quad + \pi(\tau_k, T) b_{k,-1} e^{-ct} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} e^{(c+j\omega_k)\tau_k} (1 - e^{-(c+j\omega_k)T}) \\
&\quad \left. + \pi(\tau_k, T) b_{k,0} e^{-ct} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} (e^{(c+j\omega_k)t} - e^{(c+j\omega_k)\tau_k}) \right] & (13) \\
& \quad 0 < t < T
\end{aligned}$$

where $\pi(t_1, t_2) = \begin{cases} 1, & t_1 \leq t \leq t_2 \\ 0, & \text{otherwise} \end{cases}$ is a unit pulse between t_1 and t_2

The FP filtered output $s(t)$ is detected by the photodetector which produces a current of $\mathcal{R} |s(t)|^2$ Amps. This current plus additive white postdetection thermal noise current from the amplifier is integrated by the integrator to obtain a decision variable for the threshold detector.

C. DECISION VARIABLES

The decision variable Y appearing at the integrator output consists of the signal component X and noise component N

$$Y = X + N \quad (14)$$

where

$$X = \int_0^T \mathcal{R} |s(t)|^2 dt \quad (15)$$

$$N = \int_0^T n(t) dt \quad (16)$$

We note that N is a zero mean Gaussian random variable with variance $N_0 T$. Substituting (10)-(13) into (15) we obtain the signal component X as a function of the three parameters τ_k , cT , and $\omega_k T$, which represent the effect of intersymbol interference and adjacent channel interference.

$$\begin{aligned} X = & \mathcal{R} PT b_{0,0}^2 \left[1 - \frac{2}{cT} (1 - e^{-cT}) + \frac{1}{2cT} (1 - e^{-2cT}) \right] \\ & + \mathcal{R} \frac{PT}{2cT} (1 - e^{-2cT}) \left[\sum_{i=-L_0}^{-1} b_{0,i} (e^{(i+1)cT} - e^{icT}) \right]^2 \end{aligned}$$

$$+ \mathcal{R} \frac{PT}{2cT} (1 - e^{-2cT}) \left| \sum_{k=-M/2}^{M/2} \sum_{l=-L}^{-2} b_{k,l} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} e^{(c+j\omega_k)(lT+\tau_k)} (e^{(c+j\omega_k)T} - 1) \right|^2$$

$$+ \mathcal{R} \frac{PT}{cT} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m \neq 0}}^{M/2} \frac{b_{k,-1} b_{m,-1}^* e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \left[\begin{array}{ll} c\tau_k & \omega_k = \omega_m \\ \frac{e^{j(\omega_k - \omega_m)\tau_{s-1}}}{j(\omega_k - \omega_m)/c} & \omega_k \neq \omega_m \end{array} \right]$$

$$+ \mathcal{R} \frac{PT}{cT} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m \neq 0}}^{M/2} \frac{b_{k,-1} b_{m,-1}^* e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \left[\frac{(e^{-(c+j\omega_m)\tau_s} - 1) e^{(c+j\omega_k)(-T+\tau_k)}}{1+j\frac{\omega_m}{c}} \right.$$

$$\left. + \frac{(e^{-(c-j\omega_k)\tau_s} - 1) e^{(c-j\omega_m)(-T+\tau_m)}}{1-j\frac{\omega_k}{c}} - \frac{(e^{-2c\tau_s} - 1) e^{(c+j\omega_k)(-T+\tau_k)} e^{(c-j\omega_m)(-T+\tau_m)}}{2} \right]$$

$$+ \mathcal{R} \frac{PT}{cT} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m \neq 0}}^{M/2} \frac{b_{k,-1} b_{m,-1}^* (e^{-2c\tau_g} - e^{-2cT}) e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{2(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})}$$

$$[e^{(c+j\omega_k)\tau_k} (1 - e^{-(c+j\omega_k)T}) e^{(c-j\omega_m)\tau_m} (1 - e^{-(c-j\omega_m)T})]$$

$$+ \mathcal{R} \frac{PT}{cT} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m \neq 0}}^{M/2} \frac{b_{k,0} b_{m,0}^* e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \left[\begin{array}{ll} cT - c\tau_k & \omega_k = \omega_m \\ \frac{e^{j(\omega_k - \omega_m)T} - e^{j(\omega_k - \omega_m)\tau_g}}{j(\omega_k - \omega_m)/c} & \omega_k \neq \omega_m \end{array} \right]$$

$$+ \mathcal{R} \frac{PT}{cT} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m \neq 0}}^{M/2} \frac{b_{k,0} b_{m,0}^* e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \left[\frac{e^{(c+j\omega_k)\tau_k} (e^{-(c+j\omega_m)T} - e^{-(c+j\omega_m)\tau_g})}{1+j\frac{\omega_m}{c}} \right.$$

$$\left. + \frac{e^{(c-j\omega_m)\tau_m} (e^{-(c-j\omega_k)T} - e^{-(c-j\omega_k)\tau_g})}{1-j\frac{\omega_k}{c}} - \frac{(e^{-2cT} - e^{-2c\tau_g}) e^{(c+j\omega_k)\tau_k} e^{(c-j\omega_m)\tau_m}}{2} \right]$$

$$+ \mathcal{R} \frac{2PT}{cT} Re \left\{ \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m \neq 0}}^{M/2} \frac{b_{k,-1} b_{m,0}^* e^{-j(\omega_k \tau_k - \omega_m \tau_m)} (1 - e^{-(c+j\omega_k)T}) e^{(c+j\omega_k)\tau_k}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \right.$$

$$\left. \left[\frac{e^{-(c+j\omega_m)\tau_m} - e^{-(c+j\omega_m)T}}{1+j\frac{\omega_m}{c}} - \frac{(e^{-2c\tau_m} - e^{-2cT}) e^{(c-j\omega_m)\tau_m}}{2} \right] \right\}$$

$$+ \mathcal{R} \frac{2PT}{cT} Re \left\{ \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m \neq 0}}^{M/2} \sum_{l=-L}^{-2} \frac{b_{k,l} b_{m,-1}^* e^{-j(\omega_k \tau_k - \omega_m \tau_m)} (e^{(c+j\omega_k)T} - 1) e^{(c+j\omega_k)(lT + \tau_k)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \right.$$

$$\left. \left[\frac{(1 - e^{-(c+j\omega_m)\tau_m})}{1+j\frac{\omega_m}{c}} - \frac{(1 - e^{-2c\tau_m}) e^{(c-j\omega_m)(-T + \tau_m)}}{2} \right] \right\}$$

$$+ \mathcal{R} \frac{2PT}{cT} Re \left\{ \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m \neq 0}}^{M/2} \sum_{l=-L}^{-2} \frac{e^{-j(\omega_k \tau_k - \omega_m \tau_m)} (e^{(c+j\omega_k)T} - 1) e^{(c+j\omega_k)(lT + \tau_k)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \right.$$

$$\left. \left[\frac{b_{k,l} b_{m,-1}^* (e^{-2c\tau_m} - e^{-2cT}) (1 - e^{-(c-j\omega_m)T}) e^{(c-j\omega_m)\tau_m}}{2} \right. \right.$$

$$\left. \left. + \frac{b_{k,l} b_{m,0}^* (e^{-(c+j\omega_m)\tau_m} - e^{-(c+j\omega_m)T})}{1+j\frac{\omega_m}{c}} \right. \right]$$

$$\left. \left. - \frac{b_{k,l} b_{m,0}^* (e^{-2c\tau_m} - e^{-2cT}) e^{(c-j\omega_m)\tau_m}}{2} \right] \right\}$$

$$\begin{aligned}
& + \mathcal{P} \frac{2PT}{cT} \operatorname{Re} \left\{ \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m \neq 0}}^{M/2} \frac{e^{j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \right. \\
& \left[\frac{b_{k,-1} b_{m,-1}^* (e^{-(c-j\omega_k)\tau_m} - e^{-(c-j\omega_k)\tau_k}) (1 - e^{-(c-j\omega_m)T}) e^{(c-j\omega_m)\tau_m}}{1-j\frac{\omega_k}{c}} \right. \\
& \left. - \frac{b_{k,-1} b_{m,-1}^* (e^{-2c\tau_m} - e^{-2c\tau_k}) (1 - e^{-(c-j\omega_m)T}) e^{(c+j\omega_k)(-T+\tau_k)} e^{(c-j\omega_m)\tau_m}}{2} \right. \\
& \left. + \frac{b_{k,-1} b_{m,0}^* (e^{j(\omega_k - \omega_m)\tau_k} - e^{j(\omega_k - \omega_m)\tau_m})}{j\frac{(\omega_k - \omega_m)}{c}} \right. \\
& \left. - \frac{b_{k,-1} b_{m,0}^* (e^{-(c-j\omega_k)\tau_m} - e^{-(c-j\omega_k)\tau_k}) e^{(c-j\omega_m)\tau_m}}{1-j\frac{\omega_k}{c}} \right. \\
& \left. - \frac{b_{k,-1} b_{m,0}^* (e^{-(c+j\omega_m)\tau_m} - e^{-(c+j\omega_m)\tau_k}) e^{(c+j\omega_k)(-T+\tau_k)}}{1+j\frac{\omega_m}{c}} \right. \\
& \left. + \frac{b_{k,-1} b_{m,0}^* (e^{-2c\tau_m} - e^{-2c\tau_k}) e^{(c+j\omega_k)(-T+\tau_k)} e^{(c-j\omega_m)\tau_m}}{2} \right] \}
\end{aligned}$$

(The above whole term will be zero if $\omega_k = \omega_m$).

$$+ \mathcal{P} \frac{PT b_{0,0} (1 - e^{-cT})^2}{cT} \sum_{i=-L_0}^{-1} b_{0,i} (e^{(i+1)cT} - e^{icT})$$

$$\begin{aligned}
& + \mathcal{R} \frac{2PT}{cT} \operatorname{Re} \left\{ \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \sum_{l=-L}^{-2} \frac{b_{0,0} b_{k,l} (1-e^{-cT})^2 e^{-j\omega_k \tau_k} (e^{(c+j\omega_k)T} - 1) e^{(c+j\omega_k)(lT+\tau_k)}}{2 (1+j\frac{\omega_k}{c})} \right. \\
& + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \frac{b_{0,0} b_{k,-1} e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} \left[\frac{e^{j\omega_k \tau_k} - 1}{j\frac{\omega_k}{c}} - \frac{1 - e^{-(c-j\omega_k)\tau_k}}{1-j\frac{\omega_k}{c}} - \frac{(1-e^{-c\tau_k})^2 e^{(c+j\omega_k)(-T+\tau_k)}}{2} \right] \\
& + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \frac{b_{0,0} b_{k,-1} e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} \left[\frac{(2e^{-c\tau_k} - 2e^{-cT} - e^{-2c\tau_k} + e^{-2cT})(1-e^{-(c+j\omega_k)T}) e^{(c+j\omega_k)\tau_k}}{2} \right] \\
& + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \frac{b_{0,0} b_{k,0} e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} \left[\frac{e^{j\omega_k T} - e^{j\omega_k \tau_k}}{j\frac{\omega_k}{c}} - \frac{e^{-(c-j\omega_k)\tau_k} - e^{-(c-j\omega_k)T}}{1-j\frac{\omega_k}{c}} \right. \\
& \quad \left. - \frac{(2e^{-c\tau_k} - 2e^{-cT} - e^{-2c\tau_k} + e^{-2cT}) e^{(c+j\omega_k)\tau_k}}{2} \right] \} \\
& + \mathcal{R} \frac{2PT}{cT} \sum_{i=-L_0}^{-1} b_{0,i} (e^{(i+1)cT} - e^{icT}) \\
& \operatorname{Re} \left\{ \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \sum_{l=-L}^{-2} \frac{b_{k,l} (1-e^{-2cT}) (e^{(c+j\omega_k)T} - 1) e^{(c+j\omega_k)lT} e^{c\tau_k}}{2 (1+j\frac{\omega_k}{c})} \right. \\
& + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \frac{b_{k,-1}}{1+j\frac{\omega_k}{c}} \left[\frac{e^{-j\omega_k \tau_k} - e^{-c\tau_k}}{1-j\frac{\omega_k}{c}} - \frac{(e^{c\tau_k} - e^{-c\tau_k}) e^{-(c+j\omega_k)T}}{2} \right. \\
& \quad \left. + \frac{(e^{-c\tau_k} - e^{c\tau_k} e^{-2cT})(1-e^{-(c+j\omega_k)T})}{2} \right] \\
& + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \frac{b_{k,0}}{1+j\frac{\omega_k}{c}} \left[\frac{e^{-c\tau_k} - e^{-j\omega_k \tau_k} e^{-(c-j\omega_k)T}}{1-j\frac{\omega_k}{c}} - \frac{e^{-c\tau_k} - e^{c\tau_k} e^{-2cT}}{2} \right] \} \quad (17)
\end{aligned}$$

where $\tau_s = \min(\tau_k, \tau_m)$ $\tau_g = \max(\tau_k, \tau_m)$

D. BIT ERROR PROBABILITY

For a detection threshold α and an ISI/ACI bit pattern $b = \{b_{k,l}, b_{0,i}\}; i = -L_0, \dots, -1; l = -L, \dots, 0; k = -M/2, \dots, M/2, k \neq 0$; the conditional bit error probability of the OOK signal represented by the Gaussian random variable Y in (14)-(16) is given by [Ref. 8]

$$P_e(b) = \frac{1}{2}Q\left(\frac{X_1(b)-\alpha}{\sqrt{N_0T}}\right) + \frac{1}{2}Q\left(\frac{\alpha-X_0(b)}{\sqrt{N_0T}}\right) \quad (18)$$

where $Q(X)$ is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-y^2/2} dy \quad (19)$$

and X_0 and X_1 are the values of X in (17) for $b_{0,0} = 0$ and $b_{0,0} = 1$, respectively. The average bit error probability P_e is obtained by taking the expected value of $P_e(b)$ in (18) over all bit patterns b . The minimum bit error probability is obtained by optimizing over the threshold α . In summary, given α , we calculate the following expectations:

$$P_e = \mathbb{E}_b \{P_e(b)\} \quad (20)$$

$$\begin{aligned} P_{e,\min} &= \min_{\alpha} \mathbb{E}_b \{P_e(b)\} \\ &= \min_{\alpha} \frac{1}{2^{M(L+1)+L_0}} \sum_{2^M \text{ patterns}} p(b) \end{aligned}$$

$$p(b) = \frac{1}{2} \left(\frac{1}{2\pi} \right)^M \left(\frac{1}{T} \right)^M$$

$$\begin{aligned}
& \left[\int_M \dots \int_0^{2\pi} \int_M \dots \int_0^T Q\left(\frac{\alpha - X_0(\phi_{-M/2} \dots \phi_{M/2} \tau_{-M/2} \dots \tau_{M/2})}{\sqrt{N_0 T}}\right) d\phi_{-M/2} \dots d\phi_{M/2} d\tau_{-M/2} \dots d\tau_{M/2} \right. \\
& \left. + \int_M \dots \int_0^{2\pi} \int_M \dots \int_0^T Q\left(\frac{X_1(\phi_{-M/2} \dots \phi_{M/2} \tau_{-M/2} \dots \tau_{M/2}) - \alpha}{\sqrt{N_0 T}}\right) d\phi_{-M/2} \dots d\phi_{M/2} d\tau_{-M/2} \dots d\tau_{M/2} \right]
\end{aligned}$$

(21)

III. NUMERICAL RESULTS

In this section we present the numerical results for a) bit error probability versus signal-to-noise ratio $Z = \Re P \sqrt{T/N_0}$ as a function of the normalized channel spacing (normalized to the bit rate)

$$I = \frac{\omega_k T}{2\pi k} = (\Delta f) T \quad (22)$$

where $\Delta f = \omega_k / 2\pi k$ is the equal channel spacing in Hz, and b) power penalty versus normalized channel spacing I as a function of the FP filter parameter $cT = (\pi / \sqrt{\rho})BT$ where B is the filter full width at half maximum bandwidth(FWHM). For our model to be valid we set $M = 2$, that is, we constraint the signal to be bandlimited to within two adjacent channels, thus we incorporate the degradation \downarrow the two nearest adjacent channels. We observe that for bit rates of 1 Gb/s or less, our model is valid up to $M = 10$ and very little difference is observed between $M = 2$ and $M = 10$. Also, we observe that there is little difference between $M = 2$ and $M = 4$ when $I \geq 10$ for bit rates up to 3 Gb/s. In all results we set $L_0 = 2$ and $L = 1$.

Figures 4-5 show the minimum bit error probability versus signal-to-noise ratio (Z) for $cT = 5$ and 10, respectively. Along with each curve, we also show that of the synchronous case, i.e, $\tau_k = 0$, $L_0 = 2$ and $L = 0$, and that of a single channel (SC) operation without filtering or with filtering but without ISI. In Fig. 4 we observe that a large degradation occurs due to ISI for $cT = 5$ which represents a narrowband filter. As

the FP filter bandwidth is made larger as in Fig. 5 with $cT = 10$, the *ISI* is reduced but the *ACI* increases.

In our model, we are constrained to $M = 2$ for the case under consideration. We use an FP filter with FWHM $B = 12.16$ GHz, free spectral range $FSR = 3800$ GHz, finesse $F = \pi\sqrt{\rho}/(1 - \rho) = FSR/B = 312.6$, and $c = 38.4$ GHz; $1/T = 2.56$ Gb/s, and $cT = 15$. If the power penalty related to single channel operation is to be less than 1 dB, then by Fig 6, a minimum normalized channel spacing of $I = 12$ must be used. Equivalently from [Eq. (22)], we could use $\Delta f = I/T = 12*2.56$ GHz = 30.4 GHz, (i.e. the channel spacing is a multiple of the bit rate). In this model the farthest adjacent channel for $M = 2$ is twice the channel spacing which is 60.8 GHz. This verifies the assumption $|f - f_0| < FSR/20\pi = 60.5$ GHz, where f_0 is the FP filter center frequency. This result agrees well with that in [Ref. 6; Figs. 6,9, $M/F = 0.4$, $a = 0.2$]. Thus we incorporate the degradation caused by the two nearest adjacent channels. We observe that for bit rates of 1 Gb/s or less, our model is valid up to $M = 10$, and very little difference is observed between $M = 2$ and $M = 10$. Also, we observe that there is little difference between $M = 2$ and $M = 4$ when $I \geq 10$ for bit rates up to 3 Gb/s. In all results we set $L_0 = 2$ and $L = 1$.

Figure 6 also shows the power penalty for an asynchronous dense WDM system relative to a single channel operation at the minimum bit error probability of 10^{-15} . This is the required additional signal power (dBW) for the asynchronous dense WDM system to be able to operate at the 10^{-15} bit error probability achieved in the single channel system with a SNR=12dB. The asynchronous dense WDM system is *ISI*-limited at 2.15 dB, 0.95 dB, 0.6 dB, and 0.5 dB in power penalty for $cT = 5, 10, 15$, and 20, respectively. It is

seen that for a 2.3 dB power penalty, the normalized channel spacing can be as close as $I = 6$ (i.e., a channel spacing of six times the bit rate) for $cT = 5$. If the power penalty criterion is 1 dB, the normalized channel spacing is $I \approx 12$ for $cT = 10, 15, 20$. We remark that although the exact transfer functions of the FP filter is used in [Ref. 6], a number of approximation have been made to obtain numerical results. The approximations are 1) the *ISI* is obtained by modeling FP filter as a single-pole RC filter [Ref. 6, Eqs. (4) and (36)], 2) approximating the finite integration with an infinite integration in the calculation of *ACI* [Ref. 6, Eq. (15)], and 3) the beat interference is ignored. On the other hand, the *ISI* and *ACI* in our investigation are obtained by modeling the FP filter as a single-pole RC filter, using finite integration and including the beat interference. Since the results in our investigation and in [Ref. 6] agree well, we conclude that approximations are quite valid. We also note that our results also agree well with the simulation carried out in [Ref. 1, Fig. 17].

The above numerical results shown in Figs. 4-6 are obtained with an optimized threshold setting. Figure 7 shows the power penalty for fixed threshold $\alpha = \mathcal{R}PT/2$ which is the same optimum threshold for single channel operation (midpoint between the received power for bit zero and bit one). It is seen that the performance of an asynchronous dense WDM system is quite sensitive to α for a narrow band filter. An additional 1.5 dB is observed for $cT = 5$ for $I > 8$, and 0.4 dB for $cT = 10$ for $I > 12$. Negligible degradation is observed for $cT = 15, 20$ for $I > 16$.

Figures 8-9 show the power penalty versus normalized channel spacing as a function of FP filter parameter cT for the worst-case analysis with optimal threshold and fixed threshold, respectively. The worst-case bit pattern is fixed to produce the minimum X_1 and maximum X_0 where X_1 and X_0 are the values of X in Appendix-A equation (4-21) with $b_{0,0} = 1$ and $b_{0,0} = 0$ respectively.

We observe that the power penalty for the worst-case analysis is only slightly larger than that of the exact analysis for $I > 10$ shown in Fig. 6. Similarly the power penalty for the worst-case analysis with fixed threshold is only slightly larger than that of the exact analysis with fixed threshold for $I > 10$ shown in Fig. 7. The reason for this is that for large channel spacing ($I > 10$), the *ACI* effect is small, so the *ACI* bit pattern has a small influence on the power penalty.

Figure 10 shows the normalized optimal threshold for the exact analysis shown in Fig. 6. It is observed that $\alpha \approx 0.4$ for $I > 10$. Note that the normalized optimal threshold for the single channel operation is $\alpha = 0.5$.

IV. CONCLUSIONS

We have presented a simple model for the analysis of asynchronous dense WDM systems employing an external OOK modulator. The only approximation that we use involves the modeling of the Fabry-Perot filter by a single-pole RC filter assuming the equivalent lowpass signal is bandlimited to the frequency range $|f| \leq FSR/20\pi$. This model enables us to obtain a closed form expression for the bit error probability which previously could only be obtained via numerical analysis [Ref. 6]. For an FP filter with an FSR around 3800 GHz, our model can include the *ACI* effects of two adjacent channels for bit rates up to 2.5 Gb/s. Our numerical results show that this model agrees well with that in [Ref. 6].

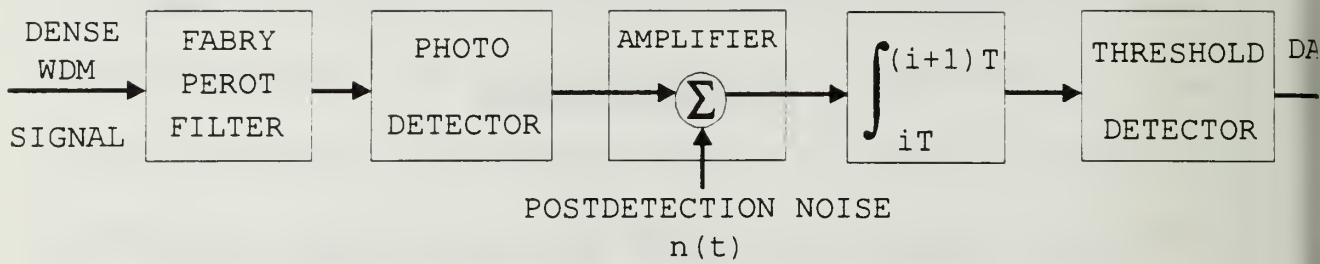


Figure 1: OOK receiver structure

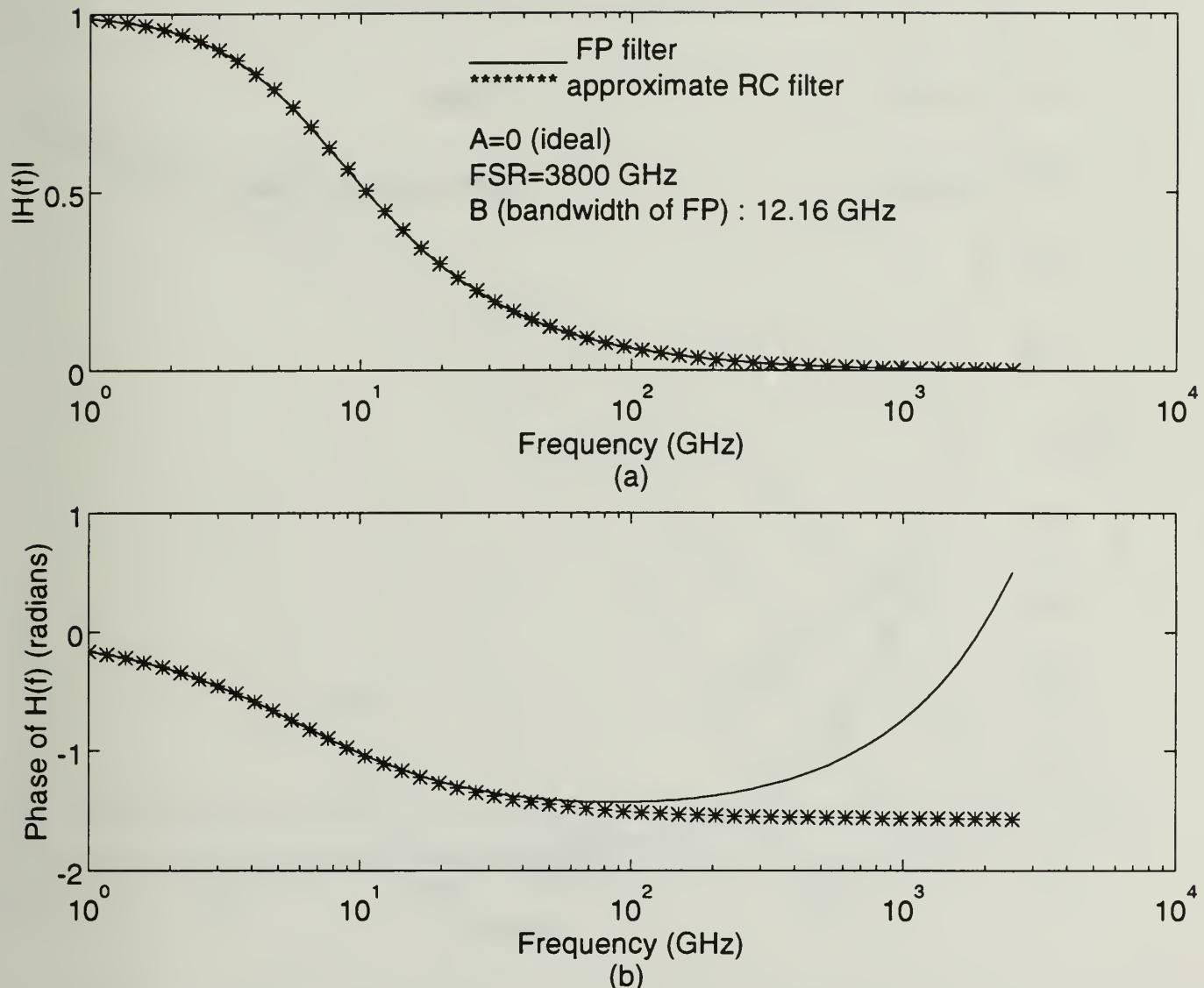


Figure 2: Spectral characteristics of the Fabry-Perot filter and the approximated single-pole RC lowpass filter

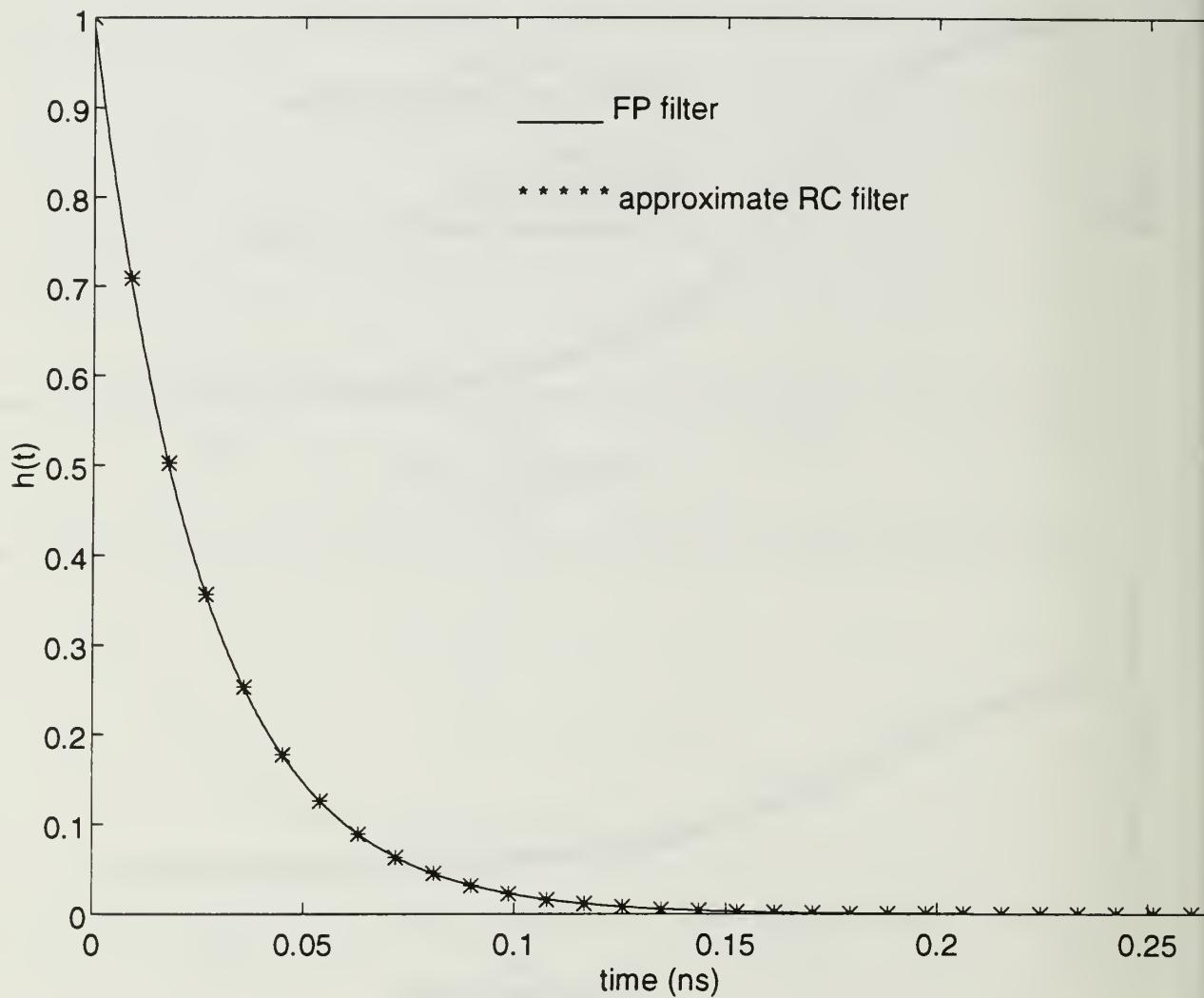


Figure 3: Normalized rimpulse response of the Fabry-Perot filter and the approximated single-pole RC lowpass filter

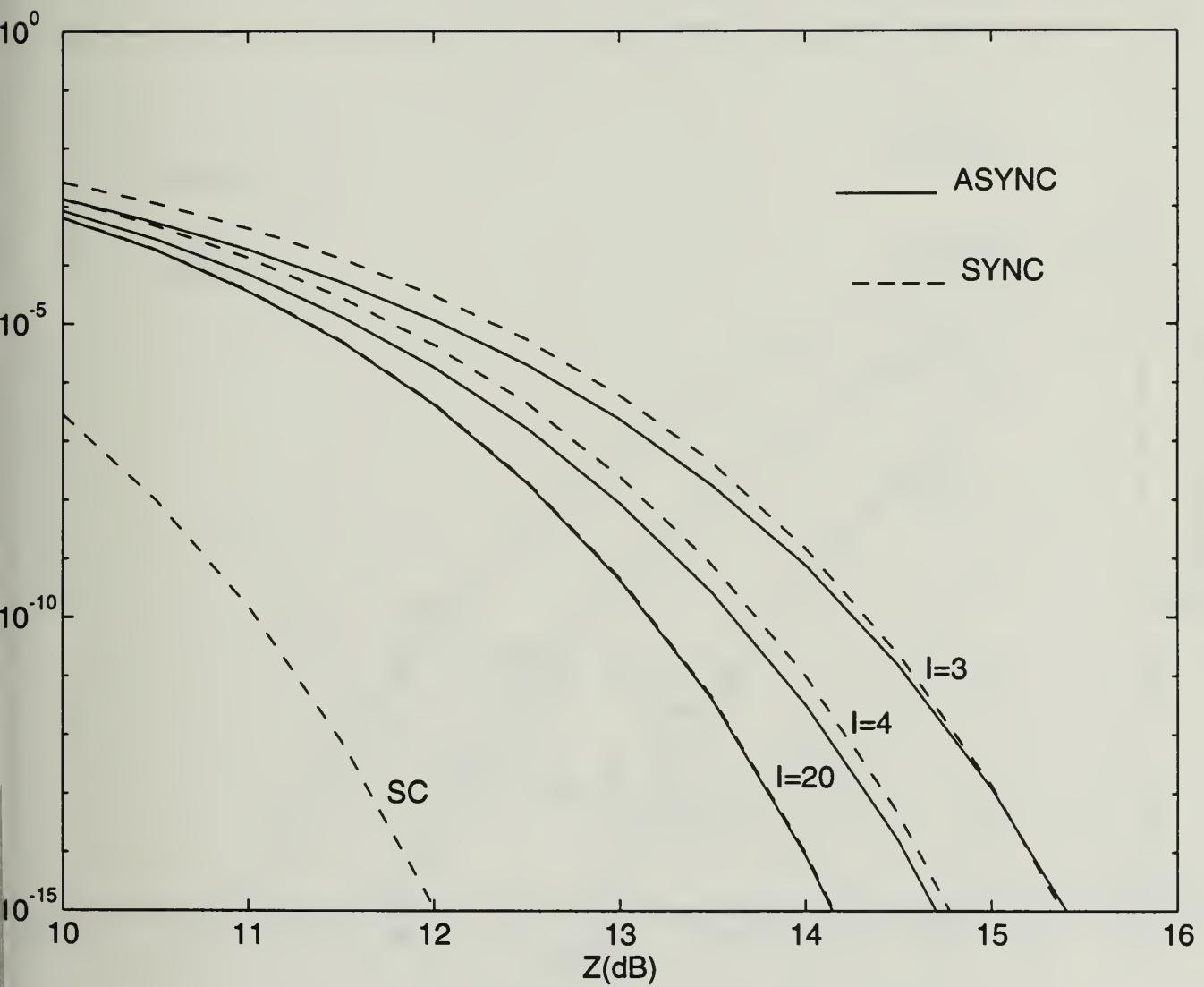


Figure 4: Probability of bit error versus signal-to-noise ratio as a function of normalized channel spacing for $cT=5$

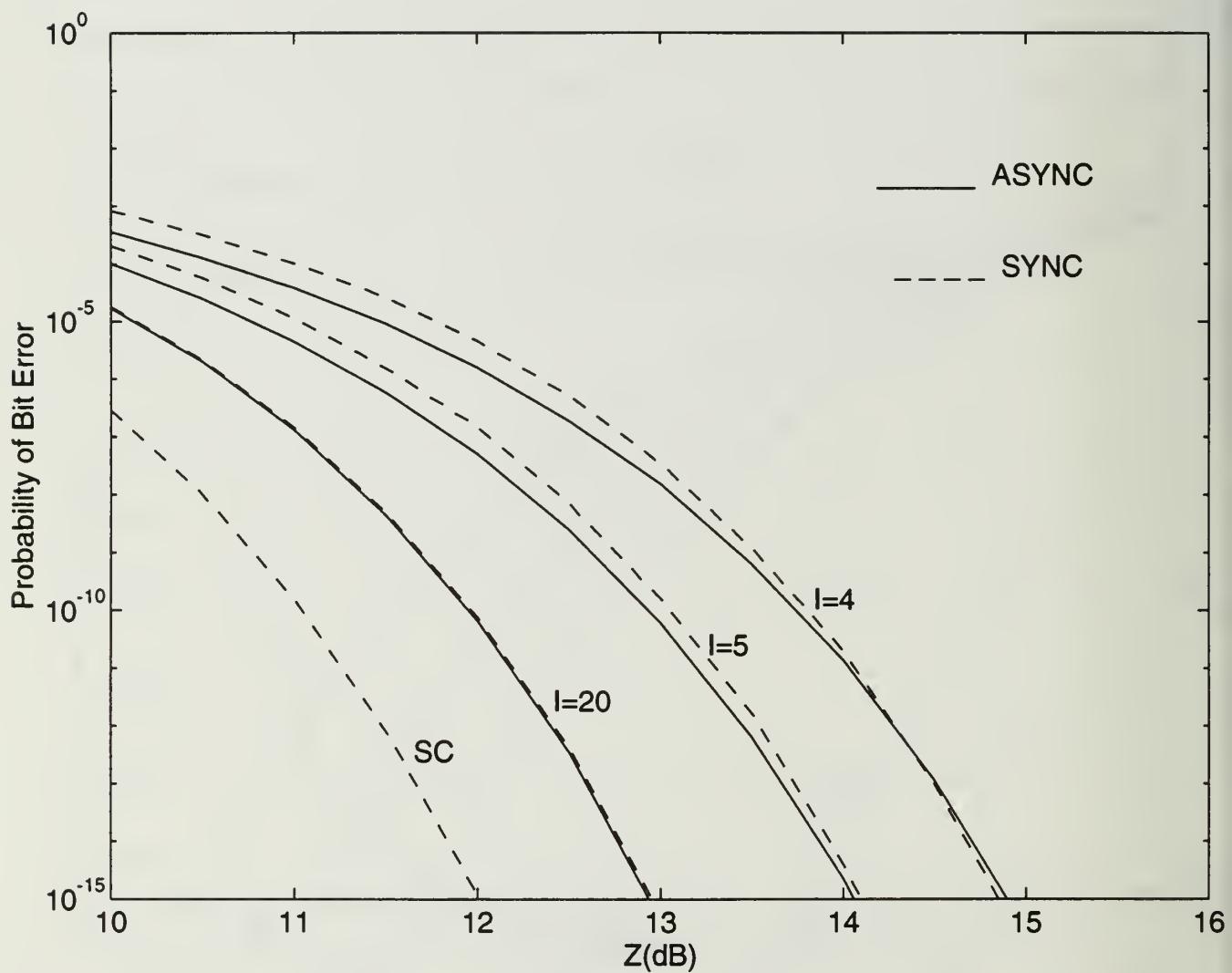


Figure 5: Probability of bit error versus signal-to-noise ratio as a function of normalized channel spacing for $cT=10$

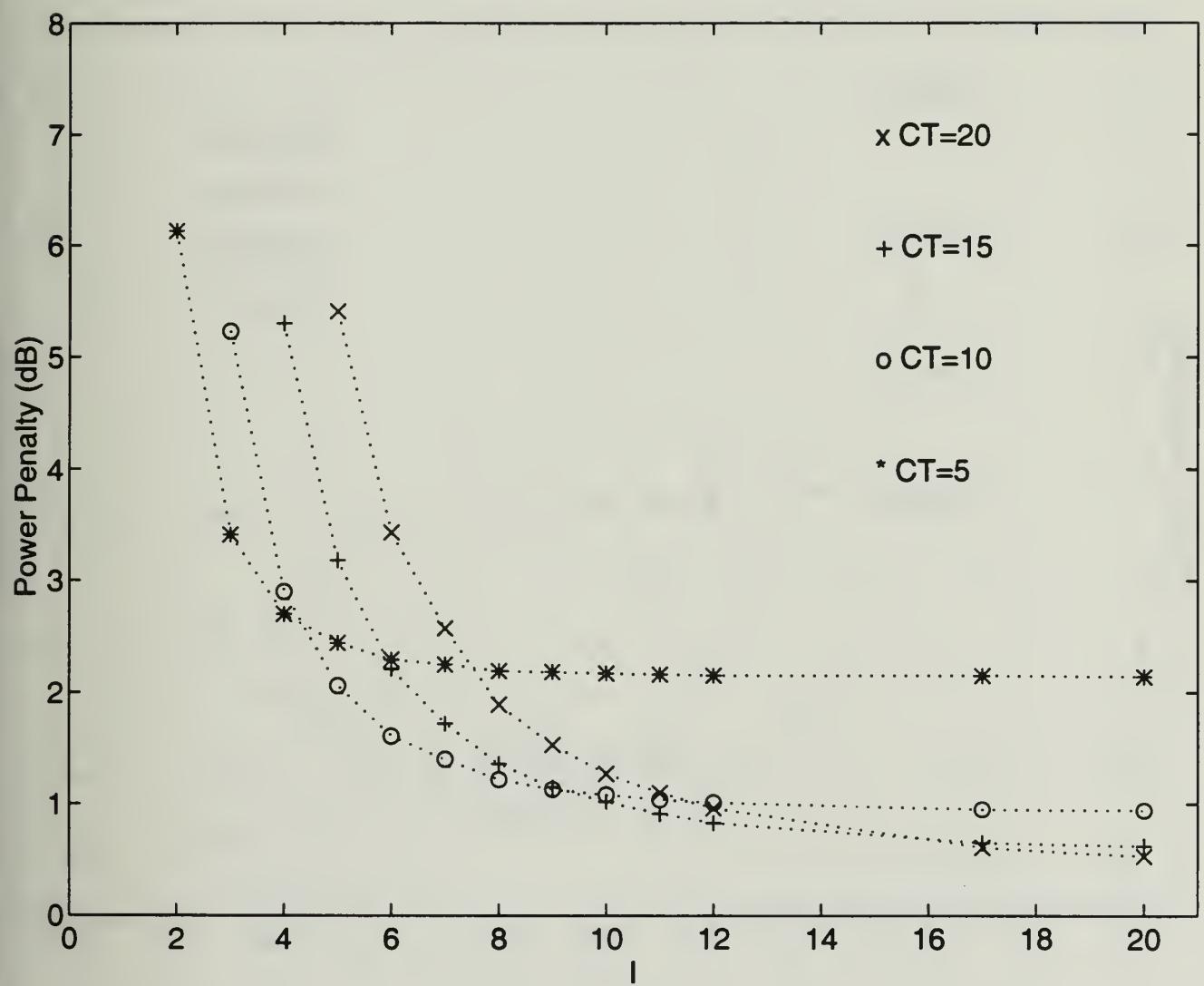


Figure 6: Power penalty versus normalized channel spacing as a function of Fabry-Perot filter parameter CT

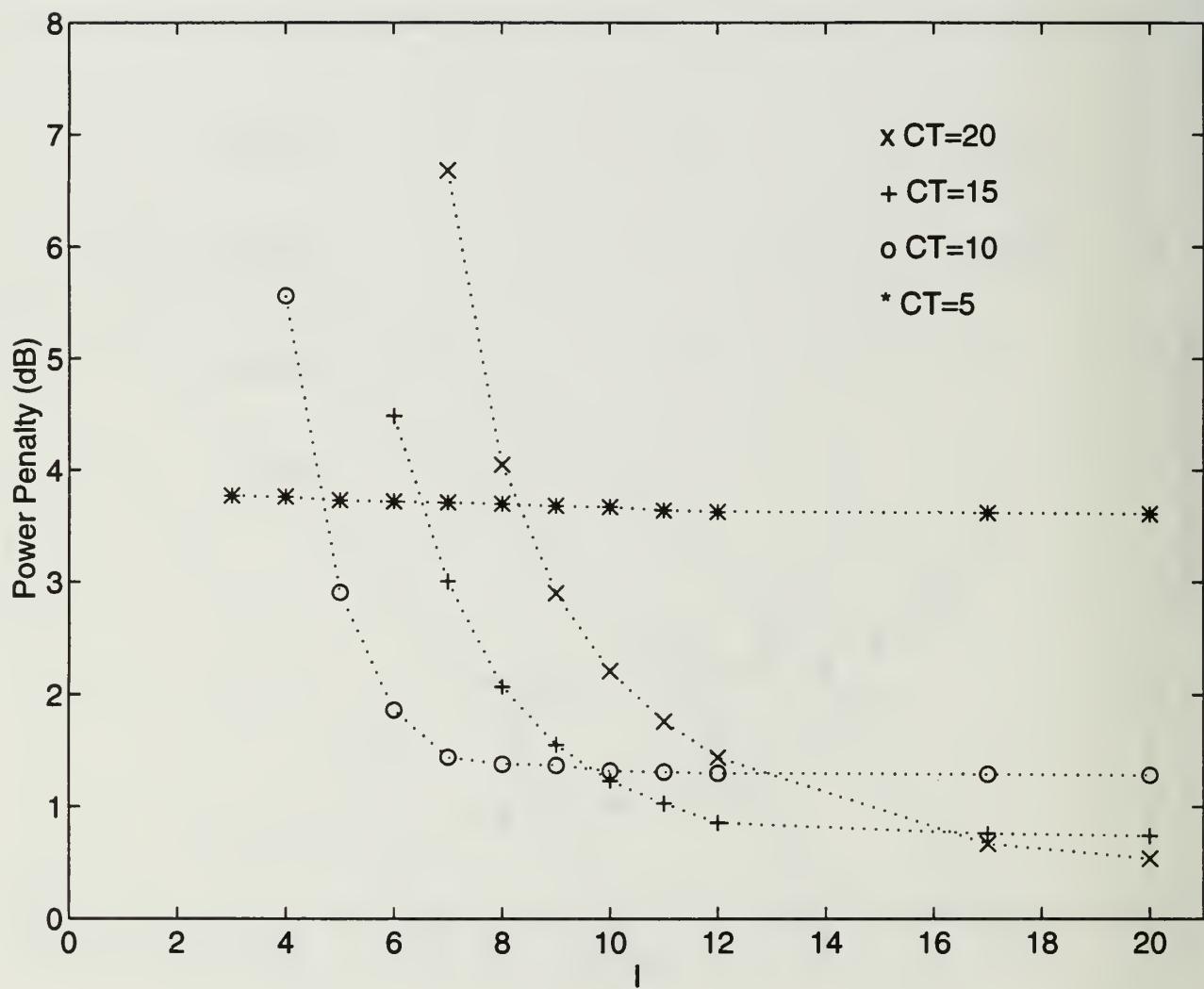


Figure 7: Power penalty versus normalized channel spacing as a function of Fabry-Perot filter parameter cT with a fixed threshold $\alpha=0.5$

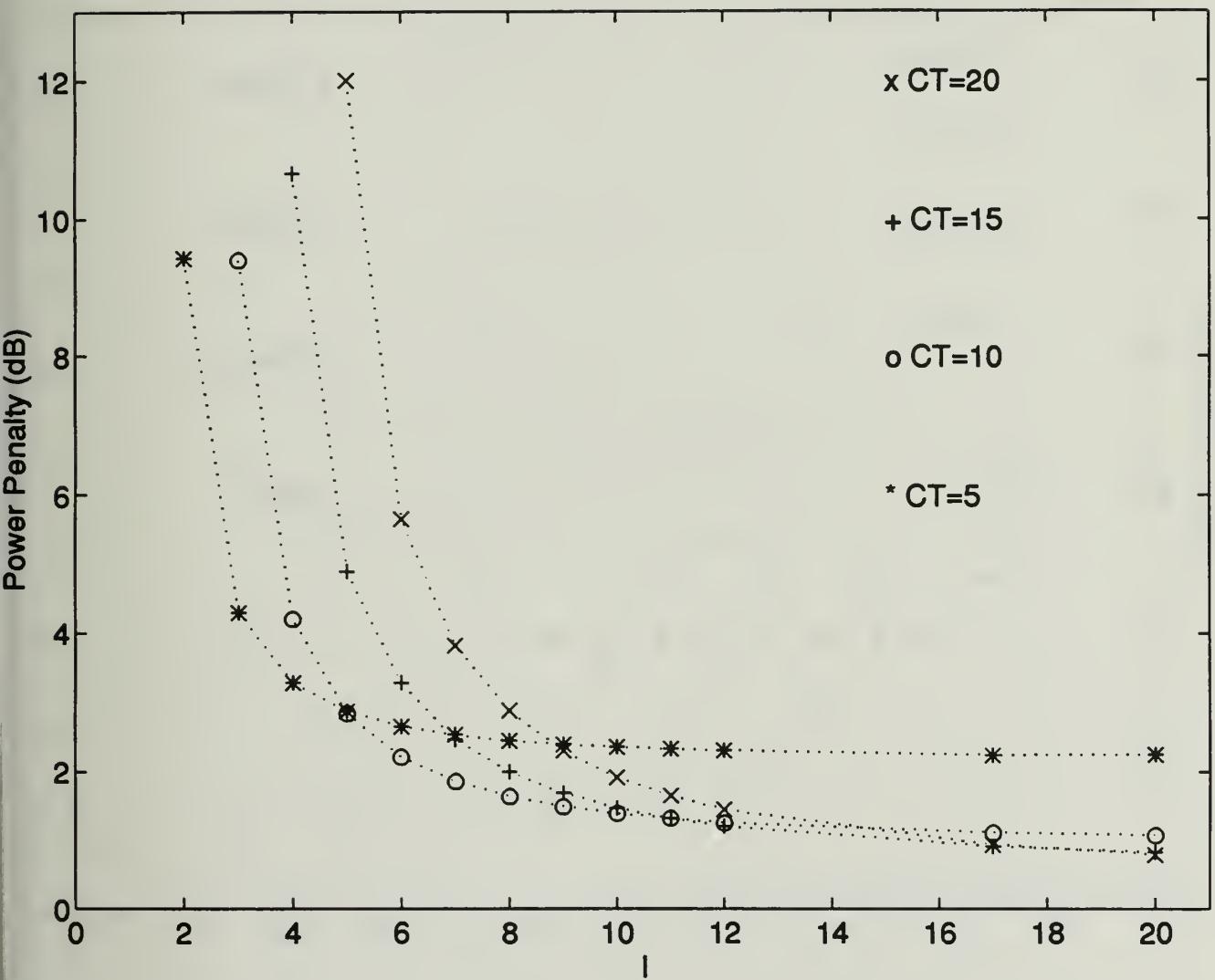


Figure 8: Power penalty versus normalized channel spacing as a function of Fabry-Perot filter parameter cT for worst-case with optimal threshold $\alpha=(X_o+X_i)/2$

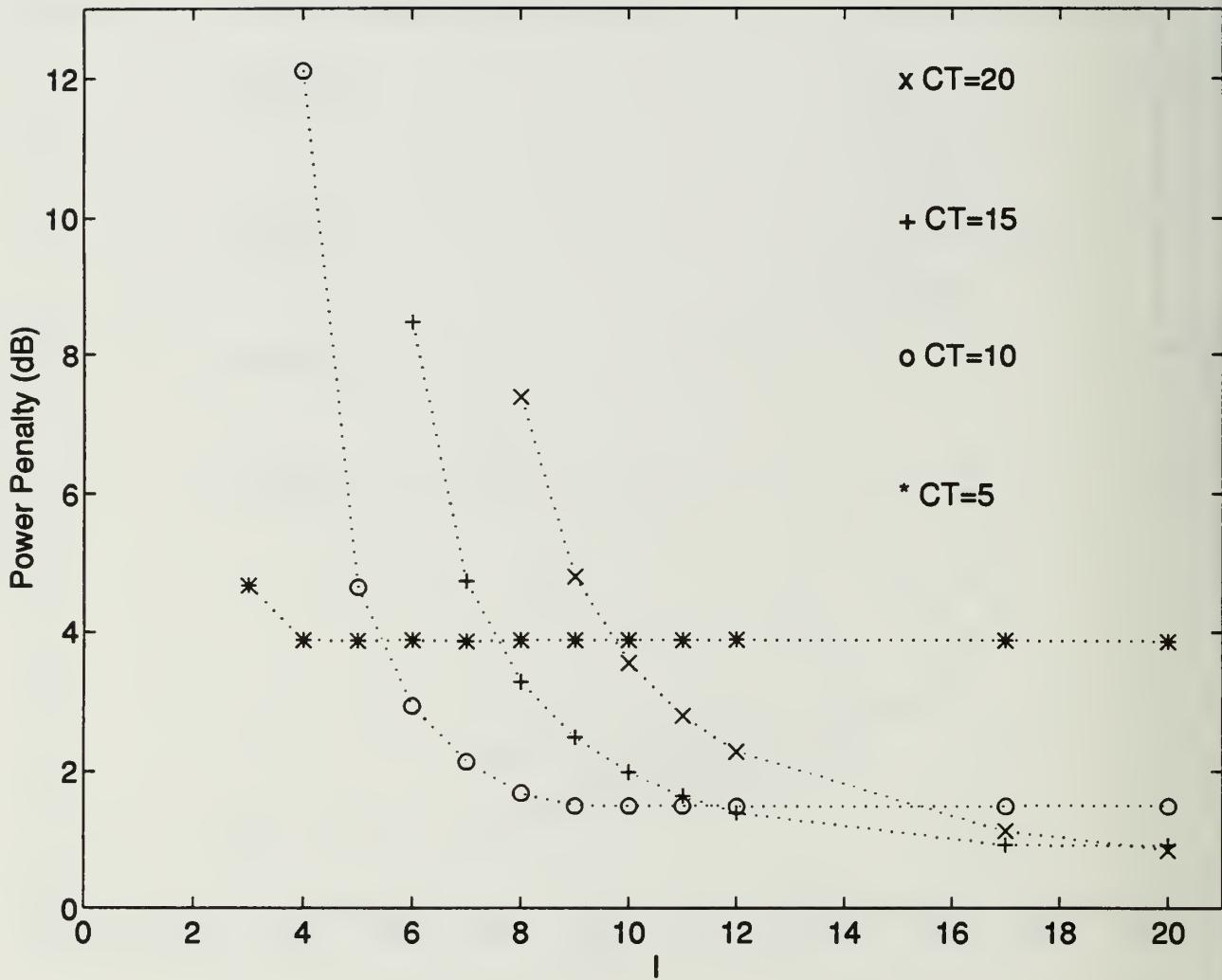


Figure 9: Power penalty versus normalized channel spacing as a function of Fabry-Perot filter parameter cT for worst-case with fixed threshold $\alpha=0.5$

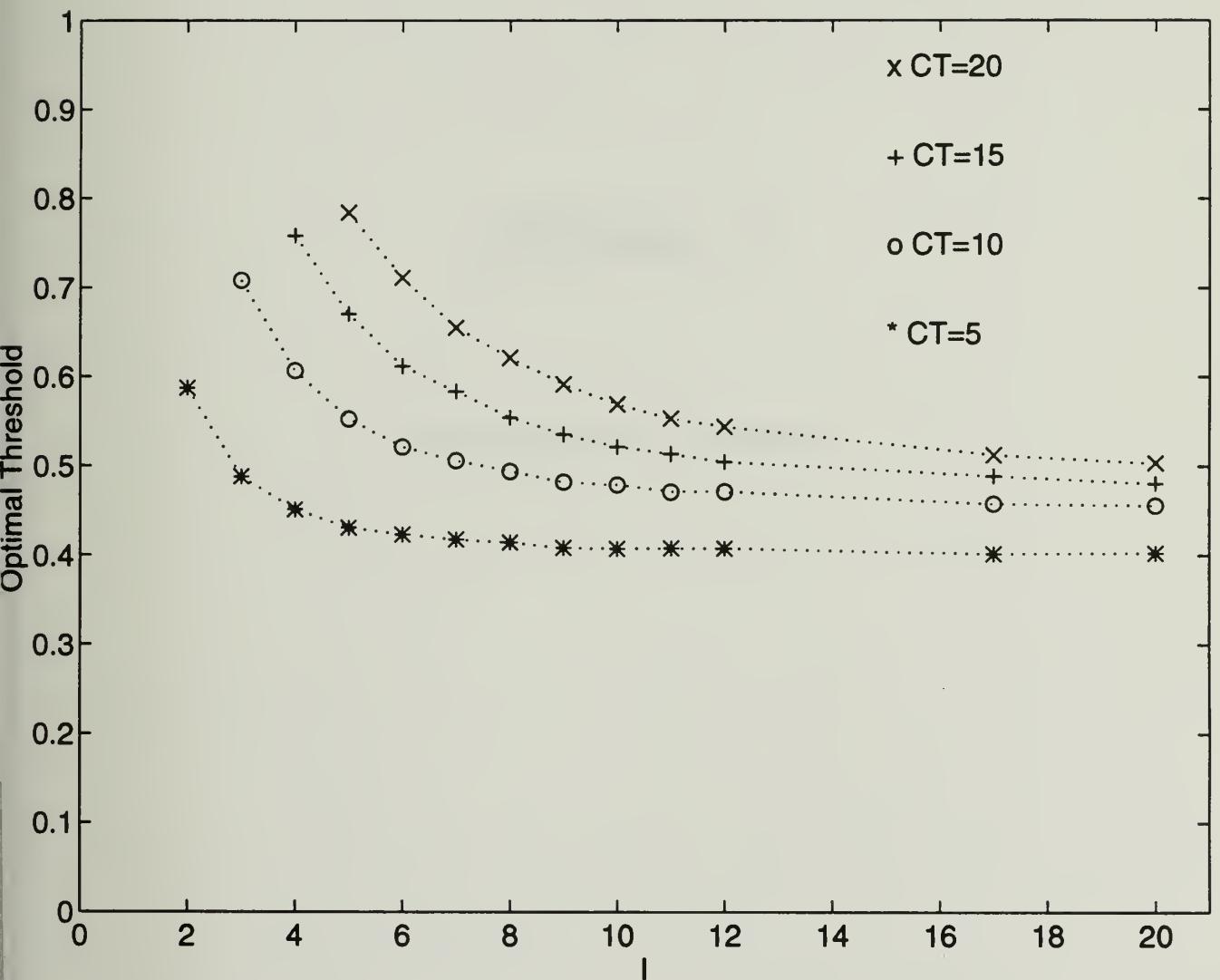


Figure 10: The normalized optimal threshold versus normalized channel spacing as a function of Fabry-Perot filter parameter cT

APPENDIX A

Derivation of Formula

I. INTRODUCTION

Desired channel : Channel 0.

Adjacent channel : Channel k .

$$k = -M/2, \dots, -1, 1, \dots, M/2, \quad M : \text{even.}$$

Bit in channel 0 in i_{th} time interval $(iT, (i+1)T) : b_{0,i} \in \{0, 1\}$

Detected bit in channel 0 in $(0, T) : b_{0,0} \in \{0, 1\}$

Bit in channel k in l_{th} time interval $(lT + \tau_k, (l+1)T + \tau_k) : b_{k,l} e^{j\omega_k(t-\tau_k)}$

$$b_{k,l} \in \{0, e^{j\phi_k}\}, \quad j = \sqrt{-1}$$

ω_k frequency spacing between channel k and channel 0, $\omega_k = -\omega_{-k}$.

ϕ_k : phase offset between channel k and channel 0, uniformly distributed between $(0, 2\pi)$.

τ_k : time delay between channel k and channel 0, $0 < \tau_k < T$.

$$\text{Data signal in channel 0 : } b_0(t) = \sum_{i=-L_0}^0 b_{0,i} P_T(t - iT) \quad (1-1)$$

$$\text{Data signal in channel } k : b_k(t) = \sum_{l=-L}^0 b_{k,l} e^{j\omega_k(t-\tau_k)} P_T(t - (lT + \tau_k)) \quad (1-2)$$

L_0, L : positive integers.

$$P_T(t) = \begin{cases} 1, & 0 \leq t \leq T \\ 0, & \text{otherwise} \end{cases}$$

The received signal at the input of the FP filter of channel 0 is :

$$r(t) = \sqrt{P} b_0(t) + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \sqrt{P} b_k(t)$$

P : received optical power.

The output of the FP filter is:

$$\begin{aligned}
r_0(t) &= \int_{-\infty}^{\infty} h(t-\lambda) r(\lambda) d\lambda \\
&= \sqrt{P} \int_{-\infty}^{\infty} h(t-\lambda) b_0(\lambda) d\lambda + \sqrt{P} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \int_{-\infty}^{\infty} h(t-\lambda) b_k(\lambda) d\lambda \\
&= \sqrt{P} b_{0,0} \int_{-\infty}^{\infty} h(t-\lambda) P_T(\lambda) d\lambda \\
&\quad + \sqrt{P} \sum_{i=-L_0}^{-1} b_{0,i} \int_{-\infty}^{\infty} h(t-\lambda) P_T(\lambda - iT) d\lambda \\
&\quad + \sqrt{P} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \left\{ \sum_{l=-L}^{-2} b_{k,l} \int_{-\infty}^{\infty} h(t-\lambda) e^{j\omega_k(\lambda-\tau_k)} P_T(\lambda - (lT + \tau_k)) d\lambda \right. \\
&\quad \left. + b_{k,-1} \int_{-\infty}^{\infty} h(t-\lambda) e^{j\omega_k(\lambda-\tau_k)} P_T(\lambda - (-T + \tau_k)) d\lambda \right. \\
&\quad \left. + b_{k,0} \int_{-\infty}^{\infty} h(t-\lambda) e^{j\omega_k(\lambda-\tau_k)} P_T(\lambda - \tau_k) d\lambda \right\} \tag{1-1}
\end{aligned}$$

$h(t)$ is the equivalent lowpass impulse response of the FP filter of channel 0. Since the detection interval is $0 < t < T$, we only need to evaluate

$S(t) = r_0(t)$, $0 < t < T$ and write $S(t)$ as :

$$S(t) = S_B(t) + S_{ISI}(t) + S_{ACI}(t) \quad 0 < t < T$$

$S_B(t)$: Desired signal in channel 0, the 1st term ($i=0$) of (1-3)

$S_{ISI}(t)$: Inter-symbol interference, the 2nd term ($i \leq -1$) of (1-3)

$S_{ACI}(t)$: Adjacent channel interference, the 3rd term (\sum_k) of (1-3)

All evaluated in $0 < t < T$.

II. CALCULATE $S_B(t)$ and $S_{ISI}(t)$

In this section we are to calculate the easy part of $S(t)$ --- $S_B(t)$ and $S_{ISI}(t)$.

A. FP filter :

The lowpass equivalent transfer function of FP filter is

$$H(f) = \frac{1-\rho}{1-\rho e^{-j2\pi f/FSR}} \cdot \frac{1-A-\rho}{1-\rho} = \frac{1-\rho}{1-\rho \cos(\frac{2\pi f}{FSR})+j\rho \sin(\frac{2\pi f}{FSR})} \cdot \frac{1-A-\rho}{1-\rho}$$

ρ : power reflectivity.

A : power absorption loss (0 for ideal filter).

FSR : free spectral range.

Since $f \ll FSR$ (for operating frequency range), we can approximate $H(f)$ as (assume $A = 0$) :

$$H(f) = \frac{1-\rho}{(1-\rho)+j\rho\frac{2\pi f}{FSR}} = \frac{1-\rho}{1+j\frac{2\pi f\rho}{(1-\rho)FSR}} = \frac{1}{1+j\frac{2\pi f}{c}}$$

$$\text{where } c = \frac{FSR(1-\rho)}{\rho}$$

for FP filter, we also have :

$$\frac{FSR}{B} = \frac{\pi\sqrt{\rho}}{1-\rho}, \text{ B : full width at half maximum bandwidth (FWHM)} \\ \text{of half power bandwidth.}$$

Using inverse Fourier transform, we find

$$h(t) = \begin{cases} ce^{-ct}, & t > 0 \\ 0, & \text{otherwise} \end{cases}$$

B. Calculate $S_B(t)$:

$$\begin{aligned}
 S_B(t) &= \sqrt{P} b_{0,0} \int_{-\infty}^{\infty} h(t-\lambda) P_T(\lambda) d\lambda \\
 &= \sqrt{P} b_{0,0} \int_0^t h(t-\lambda) d\lambda \\
 &= \sqrt{P} b_{0,0} \int_0^t c e^{-c(t-\lambda)} d\lambda \\
 &= \sqrt{P} b_{0,0} c e^{-ct} \int_0^t e^{c\lambda} d\lambda \\
 &= \sqrt{P} b_{0,0} (1 - e^{-ct}) \quad 0 < t < T
 \end{aligned} \tag{2-1}$$

$$\begin{aligned}
 \text{(a) For Bit "1" : } S_{B1}(t) &= \sqrt{P} (1 - e^{-ct}) \quad 0 < t < T \\
 \text{(b) For Bit "0" : } S_{B0}(t) &= 0 \quad 0 < t < T
 \end{aligned}$$

C. Calculate $S_{ISI}(t)$

$$\begin{aligned}
 S_{ISI}(t) &= \sqrt{P} \sum_{i=-L_0}^{-1} b_{0,i} \int_{-\infty}^{\infty} h(t-\lambda) P_T(\lambda - iT) d\lambda \\
 &= \sqrt{P} \sum_{i=-L_0}^{-1} b_{0,i} \int_{iT}^{(i+1)T} h(t-\lambda) d\lambda \\
 &= \sqrt{P} \sum_{i=-L_0}^{-1} b_{0,i} \int_{iT}^{(i+1)T} c e^{-c(t-\lambda)} d\lambda \\
 &= \sqrt{P} e^{-ct} \sum_{i=-L_0}^{-1} b_{0,i} (e^{(i+1)cT} - e^{icT}) d\lambda \quad 0 < t < T
 \end{aligned} \tag{2-2}$$

III. CALCULATE $S_{ACI}(t)$

In this section we are to find S_{ACI} with the procedure :

- A. When $l \leq -2$
- B. When $l = -1$
- C. When $l = 0$
- D. Summary for $S_{ACI}(t)$

A. When $l \leq -2$

Integrate the first term of $b_k(t)$ in (1-3), we have :

$$\begin{aligned}
 & \sqrt{P} \sum_{k=-M/2}^{M/2} \sum_{\substack{l=-2 \\ k \neq 0}}^{-2} b_{k,l} \int_{-\infty}^{\infty} h(t-\lambda) e^{j\omega_k(\lambda-\tau_k)} P_T(\lambda - (lT + \tau_k)) d\lambda \\
 &= \sqrt{P} \sum_k \sum_l^{-2} b_{k,l} \int_{lT+\tau_k}^{(l+1)T+\tau_k} h(t-\lambda) e^{j\omega_k(\lambda-\tau_k)} d\lambda \\
 &= \sqrt{P} \sum_k \sum_l^{-2} b_{k,l} \int_{lT+\tau_k}^{(l+1)T+\tau_k} ce^{-c(t-\lambda)} e^{j\omega_k(\lambda-\tau_k)} d\lambda \\
 &= \sqrt{P} \sum_k \sum_l^{-2} b_{k,l} \int_{lT+\tau_k}^{(l+1)T+\tau_k} ce^{-(ct+j\omega_k\tau_k)} e^{(c+j\omega_k)\lambda} d\lambda \\
 &= \sqrt{P} \sum_k \sum_l^{-2} \frac{b_{k,l}}{1+j\frac{\omega_k}{c}} e^{-(ct+j\omega_k\tau_k)} e^{(c+j\omega_k)(lT+\tau_k)} (e^{(c+j\omega_k)T} - 1) \\
 &= \sqrt{P} \sum_k \sum_l^{-2} b_{k,l} e^{-ct} \frac{e^{-j\omega_k\tau_k}}{1+j\frac{\omega_k}{c}} e^{(c+j\omega_k)(lT+\tau_k)} (e^{(c+j\omega_k)T} - 1) \quad 0 < t < T \quad (3-1) \\
 &= \sqrt{P} \sum_k \sum_l^{-2} \frac{b_{k,l}}{1+j\frac{\omega_k}{c}} e^{-c(t-\tau_k)} (e^{(c+j\omega_k)(lT+T)} - e^{(c+j\omega_k)lT}) \quad 0 < t < T \quad (3-1a)
 \end{aligned}$$

B. when $l = -1$

B-1 $l = -1, 0 < t < T$

Integrate the second term of $b_k(t)$ in (1-3) :

$$\begin{aligned}
 & \sqrt{P} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} b_{k,-1} \int_{-\infty}^{\infty} h(t-\lambda) e^{j\omega_k(\lambda-\tau_k)} P_T(\lambda - (-T + \tau_k)) d\lambda \\
 &= \sqrt{P} \sum_k b_{k,-1} \int_{-T+\tau_k}^t h(t-\lambda) e^{j\omega_k(\lambda-\tau_k)} d\lambda \\
 &= \sqrt{P} \sum_k b_{k,-1} \int_{-T+\tau_k}^t c e^{-(ct+j\omega_k\tau_k)} e^{(c+j\omega_k)\lambda} d\lambda \\
 &= \sqrt{P} \sum_k b_{k,-1} e^{-ct} \frac{e^{-j\omega_k\tau_k}}{1+j\frac{\omega_k}{c}} (e^{(c+j\omega_k)t} - e^{(c+j\omega_k)(-T+\tau_k)}) \quad 0 < t < \tau_k \quad (3) \\
 &= \sqrt{P} \sum_k \frac{b_{k,-1}}{1+j\frac{\omega_k}{c}} (e^{j\omega_k(t-\tau_k)} - e^{-c(t-\tau_k)} e^{-(c+j\omega_k)T}) \quad 0 < t < \tau_k
 \end{aligned}$$

B-2 $l = -1, \tau_k < t < T$

$$\begin{aligned}
 & \sqrt{P} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} b_{k,-1} \int_{-\infty}^{\infty} h(t-\lambda) e^{j\omega_k(\lambda-\tau_k)} P_T(\lambda - (-T + \tau_k)) d\lambda \\
 &= \sqrt{P} \sum_k b_{k,-1} \int_{-T+\tau_k}^{\tau_k} h(t-\lambda) e^{j\omega_k(\lambda-\tau_k)} d\lambda \\
 &= \sqrt{P} \sum_k b_{k,-1} e^{-ct} \frac{e^{-j\omega_k\tau_k}}{1+j\frac{\omega_k}{c}} e^{(c+j\omega_k)\tau_k} (1 - e^{-(c+j\omega_k)T}) \quad \tau_k < t < T \quad (3) \\
 &= \sqrt{P} \sum_k \frac{b_{k,-1}}{1+j\frac{\omega_k}{c}} e^{-c(t-\tau_k)} (1 - e^{-(c+j\omega_k)T}) \quad \tau_k < t < T \quad (3)
 \end{aligned}$$

C. when $l = 0$

C-1. $l = 0, 0 < t < \tau_k$

$$\sqrt{P} \sum_k b_{k,0} \int_{-\infty}^{\infty} h(t-\lambda) e^{j\omega_k(\lambda-\tau_k)} P_T(\lambda-\tau_k) d\lambda = 0 \quad 0 < t < \tau_k \quad (3-4)$$

C-2. $l = 0, \tau_k < t < T$

Integrate the third term of $b_k(t)$ in (1-3) :

$$\begin{aligned} & \sqrt{P} \sum_k b_{k,0} \int_{-\infty}^{\infty} h(t-\lambda) e^{j\omega_k(\lambda-\tau_k)} P_T(\lambda-\tau_k) d\lambda \\ &= \sqrt{P} \sum_k b_{k,0} \int_{\tau_k}^t h(t-\lambda) e^{j\omega_k(\lambda-\tau_k)} d\lambda \\ &= \sqrt{P} \sum_k b_{k,0} e^{-ct} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} (e^{(c+j\omega_k)t} - e^{(c+j\omega_k)\tau_k}), \quad \tau_k < t < T \end{aligned} \quad (3-5)$$

$$= \sqrt{P} \sum_k \frac{b_{k,0}}{1+j\frac{\omega_k}{c}} (e^{j\omega_k(t-\tau_k)} - e^{-c(t-\tau_k)}), \quad \tau_k < t < T \quad (3-5a)$$

D. Summary for $S_{ACI}(t)$

Define : $\pi(t_1, t_2) = U_{t_1}(t) - U_{t_2}(t), \quad t \geq 0, \quad t_1 \leq t_2$

$$\text{where } U_c(t) = \begin{cases} 0, & t < c \\ 1, & t \geq c \end{cases} \quad c \geq 0$$

is the unit step function.

$$\text{then } \pi(t_1, t_2) = \begin{cases} 1, & t_1 \leq t \leq t_2 \\ 0, & \text{otherwise} \end{cases}$$

is a unit pulse between t_1 and t_2 .

hence for $0 < t < T$ $S_{ACI}(t)$ can be written as :

$$S_{ACI}(t) = \sqrt{P} \sum_{k=-M/2}^{M/2} \sum_{\substack{l=-L \\ k \neq 0}}^{-2} \pi(0, T) b_{k,l} e^{-ct} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} e^{(c+j\omega_k)(lT+\tau_k)} (e^{(c+j\omega_k)T} - 1) \quad (3-6)$$

$$+ \sqrt{P} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \pi(0, \tau_k) b_{k,-1} e^{-ct} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} (e^{(c+j\omega_k)t} - e^{(c+j\omega_k)(-T+\tau_k)}) \quad (3-7)$$

$$+ \left[\sqrt{P} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \pi(\tau_k, T) b_{k,-1} e^{-ct} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} e^{(c+j\omega_k)\tau_k} (1 - e^{-(c+j\omega_k)T}) \right] \quad (3-8)$$

$$+ \sqrt{P} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \pi(\tau_k, T) b_{k,0} e^{-ct} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} (e^{(c+j\omega_k)t} - e^{(c+j\omega_k)\tau_k}) \quad (3-9)$$

$$= Z_0^T + Z_0^{\tau_k} + Z_{\tau_k}^T (= Z_{\tau_{k,-1}}^T + Z_{\tau_{k,0}}^T)$$

$$Z_0^T \rightarrow \quad (3-1)$$

$$Z_0^{\tau_k} \rightarrow \quad (3-2)$$

$$Z_{\tau_{k,-1}}^T \rightarrow \quad (3-3)$$

$$Z_{\tau_{k,0}}^T \rightarrow \quad (3-5)$$

IV. WORST-CASE ANALYSIS

- A. Worst-case for $S(t)$.
- B. Worst-case for $\mathcal{R} \int_0^T |S(t)|^2 dt$ - output of integrator.
- C. Special worst-case analysis.
- D. Special worst-case at the output of the integrator.

A. Worst-case for $S(t)$

A-1 Worst-case, ISI : $b_{0,i} = b_-$, $L_0 = \infty$

From (2-2), we have :

$$\begin{aligned}
 S_{ISI}^{wc}(t) &= \sqrt{P} b_- e^{-ct} (e^{cT} - 1) \sum_{i=-\infty}^{-1} e^{icT} \\
 &= \sqrt{P} b_- e^{-ct} (e^{cT} - 1) \sum_{h=1}^{\infty} (e^{-cT})^h \quad h = -i \\
 &= \sqrt{P} b_- e^{-ct} (e^{cT} - 1) \frac{e^{-cT}}{1 - e^{-cT}} \\
 &= \sqrt{P} b_- e^{-ct} (e^{cT} - 1) \frac{1}{e^{cT} - 1} \\
 &= \sqrt{P} b_- e^{-ct} \quad 0 < t < T \tag{4-1}
 \end{aligned}$$

A-2 Worst-case ACI : $b_{k,l} = b_{k,-l} = b_{k,0} = b, \quad L = \infty$

$$\begin{aligned}
S_{ACI}^{wc}(t) &= \sqrt{P} b \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \{ \pi(0, T) \frac{e^{-c(t-\tau_k)}}{1+j\frac{\omega_k}{c}} (e^{(c+j\omega_k)T} - 1) \left[\sum_{l=-\infty}^{-2} (e^{(c+j\omega_k)T})^l \right] \\
&\quad + \sqrt{P} b \sum_k \{ \pi(0, \tau_k) \frac{1}{1+j\frac{\omega_k}{c}} (e^{j\omega_k(t-\tau_k)} - e^{-c(t-\tau_k)} e^{-(c+j\omega_k)T}) \} \\
&\quad + \sqrt{P} b \sum_k \{ \pi(\tau_k, T) \frac{1}{1+j\frac{\omega_k}{c}} (e^{-c(t-\tau_k)} - e^{-c(t-\tau_k)} e^{-(c+j\omega_k)T}) \\
&\quad + \pi(\tau_k, T) \frac{1}{1+j\frac{\omega_k}{c}} (e^{j\omega_k(t-\tau_k)} - e^{-c(t-\tau_k)}) \} \\
\\
&= \sqrt{P} b \sum_k \pi(0, T) \frac{1}{1+j\frac{\omega_k}{c}} e^{-c(t-\tau_k)} e^{-(c+j\omega_k)T} \\
&\quad + \sqrt{P} b \sum_k \pi(0, \tau_k) \frac{1}{1+j\frac{\omega_k}{c}} (e^{j\omega_k(t-\tau_k)} - e^{-c(t-\tau_k)} e^{-(c+j\omega_k)T}) \\
&\quad + \sqrt{P} b \sum_k \pi(\tau_k, T) \frac{1}{1+j\frac{\omega_k}{c}} (e^{j\omega_k(t-\tau_k)} - e^{-c(t-\tau_k)} e^{-(c+j\omega_k)T}) \\
\\
&= \sqrt{P} b \sum_k \pi(0, T) \frac{1}{1+j\frac{\omega_k}{c}} (e^{-c(t-\tau_k)} e^{-(c+j\omega_k)T}) \\
&\quad + \sqrt{P} b \sum_k \pi(0, T) \frac{1}{1+j\frac{\omega_k}{c}} (e^{j\omega_k(t-\tau_k)} - e^{-c(t-\tau_k)} e^{-(c+j\omega_k)T}) \\
\\
&= \sqrt{P} b \sum_k \pi(0, T) \frac{e^{j\omega_k(t-\tau_k)}}{1+j\frac{\omega_k}{c}} \quad 0 < t < T
\end{aligned}$$

B. Worst-case for $\mathcal{R} \int_0^T |S(t)|^2 dt$ - output of integrator

Output of photodetector : $\mathcal{R} |S(t)|^2$

$$|S(t)|^2 = |S_B(t) + S_{ISI}(t) + S_{ACI}(t)|^2$$

$$\begin{aligned}
 &= |S_B(t)|^2 + |S_{ISI}(t)|^2 + |S_{ACI}(t)|^2 + 2\operatorname{Re}\{S_B(t)S_{ISI}^*(t)\} \\
 &\quad + 2\operatorname{Re}\{S_B^*(t)S_{ACI}(t)\} + 2\operatorname{Re}\{S_{ISI}^*(t)S_{ACI}(t)\} \tag{4-3}
 \end{aligned}$$

B-1 From (2-1)

$$|S_B^{wc}(t)|^2 = |S_B(t)|^2 = Pb_{0,0}^2 (1 - 2e^{-ct} + e^{-2ct}), \quad 0 < t < T, \tag{4-4}$$

$$\int_0^T |S_B(t)|^2 dt = PTb_{0,0}^2 \left[1 - \frac{2}{cT} (1 - e^{-cT}) + \frac{1}{2cT} (1 - e^{-2cT}) \right] \tag{4-5}$$

B-2 From (4-1)

$$|S_{ISI}^{wc}(t)|^2 = Pb_{-}^2 e^{-2ct}, \quad 0 < t < T, \tag{4-6}$$

$$\int_0^T |S_{ISI}^{wc}(t)|^2 dt = \frac{PTb_{-}^2}{2cT} (1 - e^{-2cT}), \tag{4-7}$$

B-3 From (4-2)

$$\begin{aligned}
 |S_{ACI}^{wc}(t)|^2 &= P|b|^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m \neq 0}}^{M/2} \frac{e^{j\omega_k(t-\tau_k)} e^{-j\omega_m(t-\tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \\
 &= P|b|^2 \sum_k \sum_m \frac{e^{j(\omega_k - \omega_m)t} e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \quad 0 < t < T,
 \end{aligned}$$

$$\begin{aligned}
 \int_0^T |S_{ACI}^{wc}(t)|^2 dt &= P|b|^2 \sum_k \sum_m \frac{e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \int_0^T e^{j(\omega_k - \omega_m)t} dt \\
 &= \begin{cases} \frac{PT|b|^2}{cT} \sum_k \sum_m \frac{e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{j(\frac{\omega_k}{c} - \frac{\omega_m}{c})(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} (e^{j(\omega_k - \omega_m)T} - 1), & \omega_k \neq \omega_m \\ PT|b|^2 \sum_k \frac{1}{1+(\frac{\omega_k}{c})^2}, & \omega_k = \omega_m \end{cases} \quad (4-2)
 \end{aligned}$$

B-4

$$2Re \{S_B(t) S_{ISI}^{wc*}(t)\} = 2Pb_{0,0}b_-(e^{-ct} - e^{-2ct}) \quad 0 < t < T \quad (4-11)$$

$$\int_0^T 2Re \{S_B(t) S_{ISI}^{wc*}(t)\} dt = \frac{PTb_{0,0}b_-}{cT} (1 - e^{-cT})^2 \quad (4-12)$$

B-5

$$\begin{aligned} 2Re \{S_B^*(t) S_{ACI}^{wc}(t)\} &= 2Pb_{0,0}Re \{b (1 - e^{-ct}) \sum_k \frac{e^{j\omega_k(t-\tau_k)}}{1+j\frac{\omega_k}{c}}\} \\ &= 2Pb_{0,0}Re \{b \sum_k \frac{e^{-j\omega_k\tau_k}}{1+j\frac{\omega_k}{c}} (e^{j\omega_k t} - e^{-(c-j\omega_k)t})\} \quad 0 < t < T \end{aligned} \quad (4-13)$$

$$\int_0^T 2Re \{S_B^*(t) S_{ACI}^{wc}(t)\} dt = \frac{2PTb_{0,0}}{cT} Re \{b \sum_k \frac{e^{-j\omega_k\tau_k}}{1+j\frac{\omega_k}{c}} \left(\frac{e^{j\omega_k T} - 1}{j\frac{\omega_k}{c}} - \frac{1 - e^{-(c-j\omega_k)T}}{1-j\frac{\omega_k}{c}} \right)\} \quad (4-14)$$

B-6

$$2Re \{S_{ISI}^{wc*}(t) S_{ACI}^{wc}(t)\} = 2Pb_-Re \{b \sum_k \frac{e^{-j\omega_k\tau_k}}{1+j\frac{\omega_k}{c}} e^{-(c-j\omega_k)t}\} \quad 0 < t < T \quad (4-15)$$

$$\int_0^T 2Re \{S_{ISI}^{wc*}(t) S_{ACI}^{wc}(t)\} dt = \frac{2PTb_-}{cT} Re \{b \sum_k \frac{e^{-j\omega_k\tau_k}}{1+(\frac{\omega_k}{c})^2} (1 - e^{-(c-j\omega_k)T})\} \quad (4-16)$$

C. Special worst-case analysis

$$\omega_k = 2\pi kI/T, \quad I : \text{positive integer}$$

$$\tau_k = 0$$

$$b_{0,i} = b_- \in \{0, 1\}$$

$$b_{k,l} = b_{k,0} = b \in \{0, e^{j\phi}\}$$

$$Re\{b\} \in \{0, \cos \phi\}$$

$$|b|^2 \in \{0, 1\}$$

$$\underline{C-1} \quad \int_0^T |S_B(t)|^2 dt = PT b_{0,0}^2 \left[1 - \frac{2}{cT} (1 - e^{-cT}) + \frac{1}{2cT} (1 - e^{-2cT}) \right] \quad (4-5)$$

$$\underline{C-2} \quad \int_0^T |S_{ISI}^{wc}(t)|_{sc}^2 dt = \frac{PTb_-^2}{2cT} (1 - e^{-2cT}) \quad (4-17)$$

$$\underline{C-3} \quad \text{in (4-9)}$$

$$\int_0^T e^{j(\omega_k - \omega_m)t} dt = \begin{cases} T, & \omega_k = \omega_m \\ 0, & \omega_k \neq \omega_m \end{cases}$$

$$\int_0^T |S_{ACI}^{wc}(t)|_{sc}^2 dt = PT|b|^2 \sum_k \frac{1}{1 + (\frac{\omega_k}{c})^2} \quad (4-18)$$

$$\underline{C-4} \quad \int_0^T 2Re \{ S_B(t) S_{ISI}^{wc*}(t) \}_{sc} dt = \frac{PTb_{0,0}b_-}{cT} (1 - e^{-cT})^2 \quad (4-12)$$

$$\underline{C-5} \quad \int_0^T 2Re \{ S_B^*(t) S_{ACI}^{wc}(t) \}_{sc} dt = \frac{2PTb_{0,0}Re\{b\}}{cT} (e^{-cT} - 1) \sum_k \frac{1}{1 + (\frac{\omega_k}{c})^2} \quad (4-19)$$

$$\underline{C-6} \quad \pm \int_0^T 2Re \{ S_{ISI}^{wc*}(t) S_{ACI}^{wc}(t) \}_{sc} dt = \frac{2PTb_-Re\{b\}}{cT} (1 - e^{-cT}) \sum_k \frac{1}{1 + (\frac{\omega_k}{c})^2} \quad (4-20)$$

D. Special worst-case at the output of the integrator

$$X_{sc}^{wc} = \mathcal{P} \int_0^T |S^{wc}(t)|_{sc}^2 dt$$

= \mathcal{P} [all terms in section C].

$$\begin{aligned}
 &= \mathcal{P}PT \left\{ b_{0,0}^2 \left[1 - \frac{2(1-e^{-cT})}{cT} + \frac{(1-e^{-2cT})}{2cT} \right] \right. \\
 &\quad + \frac{b_-^2}{2cT} (1 - e^{-2cT}) \\
 &\quad + \frac{b_{0,0} b_-}{cT} (1 - e^{-cT})^2 \\
 &\quad \left. + \left[|b|^2 + \frac{2(1-e^{-cT})}{cT} \operatorname{Re} \{b\} (b_- - b_{0,0}) \right] \sum_k \frac{1}{1 + \left(\frac{2\pi k I}{cT}\right)^2} \right\} \quad (4-21)
 \end{aligned}$$

V. EXACT ANALYSIS - GENERAL CASE

Now, we are to encounter the most complicated situation, for clarity, we first list the construction table, for which we are to follow :

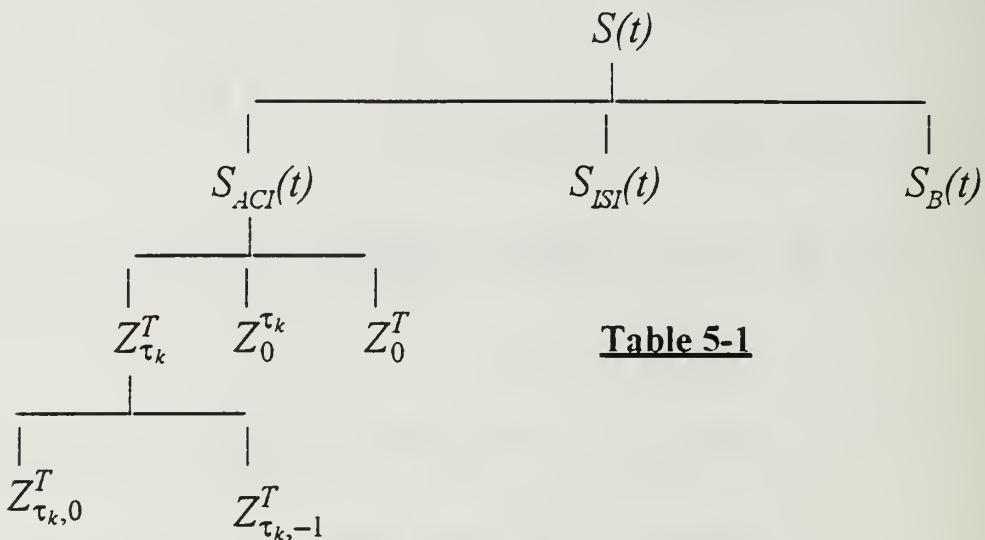


Table 5-1

$ S(t) ^2$					
F	E	D	C	B	A
$2Re\{S_{ISI}^*(t)S_{ACI}(t)\}$	$2Re\{S_B^*(t)S_{ACI}(t)\}$	$2Re\{S_B(t)S_{ISI}^*(t)\}$	$ S_{ACI}(t) ^2$	$ S_{ISI}(t) ^2$	$ S_B(t) ^2$
C-6	C-5	C-4	C-3	C-2	C-1
$2Re\{Z_0^{\tau_k}Z_{\tau_k}^{T*}\}$	$2Re\{Z_0^T Z_{\tau_k}^{T*}\}$	$2Re\{Z_0^T Z_0^{\tau_k*}\}$	$ Z_{\tau_k}^T ^2$	$ Z_0^{\tau_k} ^2$	$ Z_0^T ^2$
C-3-3	C-3-2	C-3-1			
$2Re\{Z_{\tau_{k-1}}^T Z_{\tau_{k,0}}^{T*}\}$	$ Z_{\tau_{k,0}}^T ^2$	$ Z_{\tau_{k-1}}^T ^2$			

Table 5-2

We first calculate each term in Table (5-2), then integrate it. For each term belongs to $|S_{ACI}(t)|^2$, we also discuss two cases: $\omega_k = \omega_m$ and $\omega_k \neq \omega_m$.

Since the computation is very complicated, here we just write down the results for each term, and omit the intermediate steps.

$$A. \quad \int_0^T |S_B(t)|^2 dt = PTb_{0,0}^2 \left[1 - \frac{2(1-e^{-cT})}{cT} + \frac{(1-e^{-2cT})}{2cT} \right] \quad (4-5)$$

B. *From (2-2)*

$$|S_{ISI}(t)|^2 = Pe^{-2ct} \left[\sum_{i=-L_0}^{-1} b_{0,i} (e^{(i+1)cT} - e^{icT}) \right]^2 \quad 0 < t < T \quad (5-1)$$

$$\int_0^T |S_{ISI}(t)|^2 dt = \frac{PT}{2cT} (1 - e^{-2cT}) \left[\sum_{i=-L_0}^{-1} b_{0,i} (e^{(i+1)cT} - e^{icT}) \right]^2 \quad (5-2)$$

C. Calcualte $|S_{ACI}(t)|^2$ - refer to (3-6)

C-1

$$|Z_0^T|^2 = Pe^{-2ct} \pi(0, T) \left| \sum_k \sum_l^{-2} b_{k,l} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} e^{(c+j\omega_k)(lT+\tau_k)} (e^{(c+j\omega_k)T} - 1) \right|^2 \quad (5-3)$$

$$\int_0^T |Z_0^T|^2 dt = \frac{PT}{2cT} (1 - e^{-2cT}) \left| \sum_k \sum_l^{-2} b_{k,l} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} e^{(c+j\omega_k)(lT+\tau_k)} (e^{(c+j\omega_k)T} - 1) \right|^2 \quad (5-4)$$

$$\underline{C-2} \quad \tau_s = \min(\tau_k, \tau_m)$$

$$\begin{aligned}
|Z_0^{\tau_k}|^2 &= P \sum_k \sum_m \pi(0, \tau_s) e^{-2ct} b_{k,-1} b_{m,-1}^* \frac{e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} (e^{(c+j\omega_k)t} - e^{(c+j\omega_k)(-T+\tau_k)}) \\
&\quad (e^{(c-j\omega_m)t} - e^{(c-j\omega_m)(-T+\tau_m)}) \\
&= P \sum_k \sum_m \pi(0, \tau_s) b_{k,-1} b_{m,-1}^* \frac{e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} [e^{j(\omega_k - \omega_m)t} - e^{-(c+j\omega_m)t} e^{(c+j\omega_k)(-T+\tau_k)} \\
&\quad - e^{-(c-j\omega_k)t} e^{(c-j\omega_m)(-T+\tau_m)} + e^{-2ct} e^{(c+j\omega_k)(-T+\tau_k)} e^{(c-j\omega_m)(-T+\tau_m)}] \quad (5-5)
\end{aligned}$$

$$\int_0^T |Z_0^{\tau_k}|^2 dt = \int_0^{\tau_s} |Z_0^{\tau_k}|^2 dt :$$

$$(a) \quad k \neq m, \quad \omega_k \neq \omega_m$$

$$\begin{aligned}
&= \frac{PT}{cT} \sum_k \sum_m b_{k,-1} b_{m,-1}^* \frac{e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \\
&\quad \left[\frac{1}{j(\frac{\omega_k}{c} - \frac{\omega_m}{c})} (e^{j(\omega_k - \omega_m)\tau_s} - 1) \right. \\
&\quad + \frac{1}{1+j\frac{\omega_m}{c}} (e^{-(c+j\omega_m)\tau_s} - 1) e^{(c+j\omega_k)(-T+\tau_k)} \\
&\quad + \frac{1}{1-j\frac{\omega_k}{c}} (e^{-(c-j\omega_k)\tau_s} - 1) e^{(c-j\omega_m)(-T+\tau_m)} \\
&\quad \left. - \frac{1}{2} (e^{-2c\tau_s} - 1) e^{(c+j\omega_k)(-T+\tau_k)} e^{(c-j\omega_m)(-T+\tau_m)} \right] \quad (5-6a)
\end{aligned}$$

(b) $k = m, \omega_k = \omega_m$

$$\begin{aligned}
&= \frac{PT}{cT} \sum_k \frac{|b_{k,-1}|^2}{1+(\frac{\omega_k}{c})^2} \left[c\tau_k + \frac{1}{1+j\frac{\omega_k}{c}} (e^{-(c+j\omega_k)\tau_k} - 1) e^{(c+j\omega_k)(-T+\tau_k)} \right. \\
&\quad \left. + \frac{1}{1-j\frac{\omega_k}{c}} (e^{-(c-j\omega_k)\tau_k} - 1) e^{(c-j\omega_k)(-T+\tau_k)} \right. \\
&\quad \left. - \frac{1}{2} (e^{-2c\tau_k} - 1) e^{(c+j\omega_k)(-T+\tau_k)} e^{(c-j\omega_k)(-T+\tau_k)} \right] \\
&= \frac{PT}{cT} \sum_k \frac{|b_{k,-1}|^2}{1+(\frac{\omega_k}{c})^2} \left[c\tau_k - \frac{1}{2} (1 - e^{2c\tau_k}) e^{-2cT} \right. \\
&\quad \left. + 2Re \left\{ \frac{1}{1+j\frac{\omega_k}{c}} (e^{-(c+j\omega_k)T} - e^{(c+j\omega_k)(-T+\tau_k)}) \right\} \right] \quad (5-6b)
\end{aligned}$$

C-3 3 terms for $|Z_{\tau_k}^T|$

C-3-1 $\tau_g = \max(\tau_k, \tau_m)$

$$\begin{aligned}
|Z_{\tau_{k,-1}}^T|^2 &= P \sum_k \sum_m \pi(\tau_g, T) e^{-2ct} b_{k,-1} b_{m,-1}^* \frac{e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \\
&\quad [e^{(c+j\omega_k)\tau_k} (1 - e^{-(c+j\omega_k)T}) e^{(c-j\omega_m)\tau_m} (1 - e^{-(c-j\omega_m)T})] \quad (5-7)
\end{aligned}$$

$$\int_0^T |Z_{\tau_{k,-1}}^T|^2 dt = \int_{\tau_g}^T |Z_{\tau_{k,-1}}^T|^2 dt :$$

(a) $k \neq m, \omega_k \neq \omega_m$

$$\begin{aligned}
&= \frac{PT}{cT} \sum_k \sum_m \frac{b_{k,-1} b_{m,-1}^*}{2} (e^{-2c\tau_g} - e^{-2cT}) \frac{e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \\
&\quad [e^{(c+j\omega_k)\tau_k} (1 - e^{-(c+j\omega_k)T}) e^{(c-j\omega_m)\tau_m} (1 - e^{-(c-j\omega_m)T})] \quad (5-8a)
\end{aligned}$$

(b) $k = m, \omega_k = \omega_m$

$$= \frac{PT}{2cT} \sum_k \frac{|b_{k-1}|^2}{1+(\frac{\omega_k}{c})^2} (1 - e^{2(-cT+c\tau_k)}) [1 + e^{-2cT} - 2e^{-cT} \operatorname{Re}\{e^{j\omega_k T}\}] \quad (5-8b)$$

C-3-2

$$\begin{aligned} |Z_{\tau_{k,0}}^T|^2 &= P \sum_k \sum_m \pi(\tau_g, T) b_{k,0} b_{m,0}^* \frac{e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \\ &[e^{j(\omega_k - \omega_m)t} - e^{-(c+j\omega_m)t}] e^{(c+j\omega_k)\tau_k} - e^{-(c-j\omega_k)t} e^{(c-j\omega_m)\tau_m} \\ &+ e^{-2ct} e^{(c+j\omega_k)\tau_k} e^{(c-j\omega_m)\tau_m} \end{aligned} \quad (5-9)$$

$$\int_0^T |Z_{\tau_{k,0}}^T|^2 dt = \int_{\tau_g}^T |Z_{\tau_{k,0}}^T|^2 dt :$$

(a) $k \neq m, \omega_k \neq \omega_m$

$$\begin{aligned} &= \frac{PT}{cT} \sum_k \sum_m b_{k,0} b_{m,0}^* \frac{e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \\ &[\frac{1}{j(\frac{\omega_k}{c} - \frac{\omega_m}{c})} (e^{j(\omega_k - \omega_m)T} - e^{j(\omega_k - \omega_m)\tau_g}) \\ &+ \frac{e^{(c+j\omega_k)\tau_k}}{1+j\frac{\omega_m}{c}} (e^{-(c+j\omega_m)T} - e^{-(c+j\omega_m)\tau_g}) \\ &+ \frac{e^{(c-j\omega_m)\tau_m}}{1-j\frac{\omega_k}{c}} (e^{-(c-j\omega_k)T} - e^{-(c-j\omega_k)\tau_g}) \\ &- \frac{1}{2} (e^{-2cT} - e^{-2c\tau_g}) e^{(c+j\omega_k)\tau_k} e^{(c-j\omega_m)\tau_m}] \end{aligned} \quad (5-10a)$$

(b) $k = m, \omega_k = \omega_m$

$$\begin{aligned} &= \frac{PT}{cT} \sum_k \frac{|b_{k,0}|^2}{1+(\frac{\omega_k}{c})^2} \cdot [cT - c\tau_k + \frac{1}{2}(1 - e^{2(-cT+c\tau_k)}) \\ &+ 2\operatorname{Re}\{ \frac{e^{(c+j\omega_k)\tau_k}}{1+j\frac{\omega_k}{c}} (e^{-(c+j\omega_k)T} - e^{-(c+j\omega_k)\tau_k}) \}] \end{aligned} \quad (5-10b)$$

C-3-3

$$\begin{aligned}
2Re\{Z_{\tau_{k,-1}}^T Z_{\tau_{k,0}}^{T^*}\} &= 2PRe\left\{\sum_k \sum_m \pi(\tau_g, T) b_{k,-1} b_{m,0}^* \frac{e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})}\right. \\
&\quad \left. [e^{-(c+j\omega_m)t} e^{(c+j\omega_k)\tau_k} (1 - e^{-(c+j\omega_k)T}) \right. \\
&\quad \left. - e^{-2ct} e^{(c-j\omega_m)\tau_m} e^{(c+j\omega_k)\tau_k} (1 - e^{-(c+j\omega_k)T})] \right\} \quad (5-11)
\end{aligned}$$

$$\begin{aligned}
\int_0^T 2Re\{Z_{\tau_{k,-1}}^T Z_{\tau_{k,0}}^{T^*}\} dt &= \int_{\tau_g}^T 2Re\{Z_{\tau_{k,-1}}^T Z_{\tau_{k,0}}^{T^*}\} dt, \quad \text{for } k \neq m \text{ and } k = m \\
&= \frac{2PT}{cT} Re\left\{\sum_k \sum_m b_{k,-1} b_{m,0}^* \frac{e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \cdot (1 - e^{-(c+j\omega_k)T}) e^{(c+j\omega_k)\tau_k} \right. \\
&\quad \left. \left[\frac{1}{(1+j\frac{\omega_m}{c})} (e^{-(c+j\omega_m)\tau_g} - e^{-(c+j\omega_m)T}) \right. \right. \\
&\quad \left. \left. - \frac{1}{2} (e^{-2c\tau_g} - e^{-2cT}) e^{(c-j\omega_m)\tau_m} \right] \right\} \quad (5-12)
\end{aligned}$$

C-4

$$\begin{aligned}
2Re\{Z_0^T Z_0^{\tau_k^*}\} &= 2PRe\left\{\sum_k \sum_m \sum_{l=-L}^{-2} \pi(0, \tau_m) b_{k,l} b_{m,-1}^* \frac{e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{C})(1-j\frac{\omega_m}{C})}\right. \\
&\quad \left[e^{-(c+j\omega_m)t} e^{(c+j\omega_k)(lT+\tau_k)} (e^{(c+j\omega_k)T} - 1) \right. \\
&\quad \left. - e^{-2ct} e^{(c-j\omega_m)(-T+\tau_m)} e^{(c+j\omega_k)(lT+\tau_k)} (e^{(c+j\omega_k)T} - 1) \right] \} \quad (5-1)
\end{aligned}$$

$$\int_0^T 2Re\{Z_0^T Z_0^{\tau_k^*}\} dt = \int_0^{\tau_m} 2Re\{Z_0^T Z_0^{\tau_k^*}\} dt \quad \text{for } k \neq m \text{ and } k = m$$

$$\begin{aligned}
&= \frac{2PT}{cT} Re\left\{\sum_k \sum_m \sum_{l=-L}^{-2} b_{k,l} b_{m,-1}^* \frac{e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} (e^{(c+j\omega_k)T} - 1) e^{(c+j\omega_k)(lT+\tau_k)} \right. \\
&\quad \left[\frac{(1-e^{-(c+j\omega_m)\tau_m}}{1+j\frac{\omega_m}{c}} - \frac{(1-e^{-2ct_m})}{2} e^{(c-j\omega_m)(-T+\tau_m)} \right] \} \quad (5-1)
\end{aligned}$$

$$\begin{aligned}
2Re\{Z_0^T Z_{\tau_k}^{T^*}\} &= 2PRe\left\{\sum_k \sum_m \sum_l^{-2} \pi(\tau_m, T) \frac{e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} (e^{(c+j\omega_k)T} - 1) e^{(c+j\omega_k)(lT+\tau_k)} \right. \\
&\quad [b_{k,l} b_{m,-1}^* e^{-2ct} e^{(c-j\omega_m)\tau_m} (1 - e^{-(c-j\omega_m)T}) \\
&\quad + b_{k,l} b_{m,0}^* e^{-(c+j\omega_m)t} \\
&\quad \left. - b_{k,l} b_{m,0}^* e^{-2ct} e^{(c-j\omega_m)\tau_m} \right] \} \quad (5-15)
\end{aligned}$$

$$\begin{aligned}
\int_0^T 2Re\{Z_0^T Z_{\tau_k}^{T^*}\} dt &= \int_{\tau_m}^T 2Re\{Z_0^T Z_{\tau_k}^{T^*}\} dt \\
&= \frac{2PT}{cT} Re\left\{\sum_k \sum_m \sum_{l=-L}^{-2} \frac{e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} (e^{(c+j\omega_k)T} - 1) e^{(c+j\omega_k)(lT+\tau_k)} \right. \\
&\quad [\frac{b_{k,l} b_{m,-1}^*}{2} (e^{-2c\tau_m} - e^{-2cT}) (1 - e^{-(c-j\omega_m)T}) e^{(c-j\omega_m)\tau_m} \\
&\quad + \frac{b_{k,l} b_{m,0}^*}{1+j\frac{\omega_m}{c}} (e^{-(c+j\omega_m)\tau_m} - e^{-(c+j\omega_m)T}) \\
&\quad \left. - \frac{b_{k,l} b_{m,0}^*}{2} (e^{-2c\tau_m} - e^{-2cT}) e^{(c-j\omega_m)\tau_m} \right] \} \quad (5-16)
\end{aligned}$$

C-6

$2Re\{Z_0^{\tau_k}Z_{\tau_k}^{T*}\}$ The product is only survived when $\tau_m < t < \tau_k$

$$\begin{aligned}
 &= 2PRe\left\{\sum_k \sum_m \pi(\tau_m, \tau_k) \frac{e^{-j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \cdot \right. \\
 &\quad [b_{k,-1} b_{m,-1}^* e^{-(c-j\omega_k)t} e^{(c-j\omega_m)\tau_m} (1 - e^{-(c-j\omega_m)T}) \\
 &\quad - b_{k,-1} b_{m,-1}^* e^{-2ct} e^{(c+j\omega_k)(-T+\tau_k)} e^{(c-j\omega_m)\tau_m} (1 - e^{-(c-j\omega_m)T}) \\
 &\quad + b_{k,-1} b_{m,0}^* e^{j(\omega_k - \omega_m)t} \\
 &\quad - b_{k,-1} b_{m,0}^* e^{-(c-j\omega_k)t} e^{(c-j\omega_m)\tau_m} \\
 &\quad \left. - b_{k,-1} b_{m,0}^* e^{-(c+j\omega_m)t} e^{(c+j\omega_k)(-T+\tau_k)} \right. \\
 &\quad \left. + b_{k,-1} b_{m,0}^* e^{-2ct} e^{(c+j\omega_k)(-T+\tau_k)} e^{(c-j\omega_m)\tau_m} \right] \}
 \end{aligned}$$

(5-17)

$$\begin{aligned}
& \int_0^T 2 \operatorname{Re} \{ Z_0^{\tau_k} Z_{\tau_k}^{T^*} \} dt = \int_{\tau_m}^{\tau_k} 2 \operatorname{Re} \{ Z_0^{\tau_k} Z_{\tau_k}^{T^*} \} dt \\
& = \frac{2PT}{cT} \operatorname{Re} \left\{ \sum_k \sum_m \frac{e^{j(\omega_k \tau_k - \omega_m \tau_m)}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} \cdot \right. \\
& \quad \left[\frac{b_{k,-1} b_{m,-1}^*}{1-j\frac{\omega_k}{c}} (e^{-(c-j\omega_k)\tau_m} - e^{-(c-j\omega_k)\tau_k}) (1 - e^{-(c-j\omega_m)T}) e^{(c-j\omega_m)\tau_m} \right. \\
& \quad \left. - \frac{b_{k,-1} b_{m,-1}^*}{2} (e^{-2c\tau_m} - e^{-2c\tau_k}) (1 - e^{-(c-j\omega_m)T}) e^{(c+j\omega_k)(-T+\tau_k)} e^{(c-j\omega_m)\tau_m} \right. \\
& \quad \left. + \frac{b_{k,-1} b_{m,0}^*}{j(\frac{\omega_k}{c} - \frac{\omega_m}{c})} (e^{j(\omega_k - \omega_m)\tau_k} - e^{j(\omega_k - \omega_m)\tau_m}) \right. \\
& \quad \left. - \frac{b_{k,-1} b_{m,0}^*}{1-j\frac{\omega_k}{c}} (e^{-(c-j\omega_k)\tau_m} - e^{-(c-j\omega_k)\tau_k}) e^{(c-j\omega_m)\tau_m} \right. \\
& \quad \left. - \frac{b_{k,-1} b_{m,0}^*}{1+j\frac{\omega_m}{c}} (e^{-(c+j\omega_m)\tau_m} - e^{-(c+j\omega_m)\tau_k}) e^{(c+j\omega_k)(-T+\tau_k)} \right. \\
& \quad \left. + \frac{b_{k,-1} b_{m,0}^*}{2} (e^{-2c\tau_m} - e^{-2c\tau_k}) e^{(c+j\omega_k)(-T+\tau_k)} e^{(c-j\omega_m)\tau_m} \right] \} \quad (5-18)
\end{aligned}$$

$$\begin{aligned}
D. \quad & \int_0^T 2 \operatorname{Re} \{ S_B(t) S_{ISI}^*(t) \} dt = \int_0^T 2 S_B(t) S_{ISI}(t) dt \\
& = 2P b_{0,0} \sum_{i=-L_0}^{-1} b_{0,i} (e^{(i+1)cT} - e^{icT}) \int_0^T (e^{-ct} - e^{-2ct}) dt \\
& = \frac{PT}{cT} b_{0,0} (1 - e^{-cT})^2 \sum_{i=-L_0}^{-1} b_{0,i} (e^{(i+1)cT} - e^{icT}) \quad (5-19)
\end{aligned}$$

$$E. \quad 2Re\{S_B^*(t) S_{ACI}(t)\}$$

$$\begin{aligned}
&= 2PRe\left\{\sum_k \sum_l^{-2} \pi(0, T)(e^{-ct} - e^{-2ct}) b_{0,0} b_{k,l} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} (e^{(c+j\omega_k)T} - 1) e^{(c+j\omega_k)(lT+\tau_k)} \right. \\
&+ \sum_k \pi(0, \tau_k) b_{0,0} b_{k,-1} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} [e^{j\omega_k t} - e^{-(c-j\omega_k)t} - (e^{-ct} - e^{-2ct}) e^{(c+j\omega_k)(-T+\tau_k)}] \\
&+ \sum_k \pi(\tau_k, T) b_{0,0} b_{k,-1} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} (e^{-ct} - e^{-2ct}) (1 - e^{-(c+j\omega_k)T}) e^{(c+j\omega_k)\tau_k} \\
&\left. + \sum_k \pi(\tau_k, T) b_{0,0} b_{k,0} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} [e^{j\omega_k t} - e^{-(c-j\omega_k)t} - (e^{-ct} - e^{-2ct}) e^{(c+j\omega_k)\tau_k}] \right\} \quad (5-20)
\end{aligned}$$

$$\int_0^T 2Re\{S_B^*(t) S_{ACI}(t)\} dt$$

$$\begin{aligned}
&= \frac{2PT}{cT} Re\left\{\sum_k \sum_l^{-2} \frac{b_{0,0} b_{k,l} (1-e^{-cT})^2}{2} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} (e^{(c+j\omega_k)T} - 1) e^{(c+j\omega_k)(lT+\tau_k)} \right. \\
&+ \sum_k b_{0,0} b_{k,-1} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} \left[\left(\frac{e^{j\omega_k \tau_k} - 1}{j\frac{\omega_k}{c}} \right) - \left(\frac{1 - e^{-(c-j\omega_k)\tau_k}}{1-j\frac{\omega_k}{c}} \right) - \frac{(1-e^{-c\tau_k})^2}{2} \right] e^{(c+j\omega_k)(-T+\tau_k)} \\
&+ \sum_k b_{0,0} b_{k,-1} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} \left[\left(\frac{2e^{-c\tau_k} - 2e^{-cT} - e^{-2c\tau_k} + e^{-2cT}}{2} \right) (1 - e^{-(c+j\omega_k)T}) e^{(c+j\omega_k)\tau_k} \right] \\
&+ \sum_k b_{0,0} b_{k,0} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} \left[\left(\frac{e^{j\omega_k T} - e^{j\omega_k \tau_k}}{j\frac{\omega_k}{c}} \right) - \left(\frac{e^{-(c-j\omega_k)\tau_k} - e^{-(c-j\omega_k)T}}{1-j\frac{\omega_k}{c}} \right) \right. \\
&\left. - \left(\frac{2e^{-c\tau_k} - 2e^{-cT} - e^{-2c\tau_k} + e^{-2cT}}{2} \right) e^{(c+j\omega_k)\tau_k} \right] \} \quad (5-21)
\end{aligned}$$

$$F. \quad 2Re \{ S_{ISI}^*(t) S_{ACI}(t) \}$$

$$\begin{aligned}
&= 2P \left[\sum_{i=-L_0}^{-1} b_{0,i} (e^{(i+1)cT} - e^{icT}) \right] \\
&\quad Re \left\{ \sum_k \sum_{l=-L}^{-2} \pi(0, T) b_{k,l} e^{-2ct} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} (e^{(c+j\omega_k)T} - 1) e^{(c+j\omega_k)(lT+\tau_k)} \right. \\
&\quad + \sum_k \pi(0, \tau_k) b_{k,-1} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} (e^{-(c-j\omega_k)t} - e^{-2ct} e^{(c+j\omega_k)(-T+\tau_k)}) \\
&\quad + \sum_k \pi(\tau_k, T) b_{k,-1} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} e^{-2ct} (1 - e^{-(c+j\omega_k)T}) e^{(c+j\omega_k)\tau_k} \\
&\quad \left. + \sum_k \pi(\tau_k, T) b_{k,0} \frac{e^{-j\omega_k \tau_k}}{1+j\frac{\omega_k}{c}} (e^{-(c-j\omega_k)t} - e^{-2ct} e^{(c+j\omega_k)\tau_k}) \right\} \tag{5-22}
\end{aligned}$$

$$\int_0^T 2Re \{ S_{ISI}^*(t) S_{ACI}(t) \} dt$$

$$\begin{aligned}
&= \frac{2PT}{cT} \sum_{i=-L_0}^{-1} b_{0,i} (e^{(i+1)cT} - e^{icT}) Re \left\{ \sum_k \sum_{l=1}^{-2} \frac{b_{k,l}(1-e^{-2cT})}{2(1+j\frac{\omega_k}{c})} (e^{(c+j\omega_k)T} - 1) e^{(c+j\omega_k)lT} e^{c\tau_k} \right. \\
&\quad + \sum_k \frac{b_{k,-1}}{1+j\frac{\omega_k}{c}} \left[\left(\frac{e^{-j\omega_k \tau_k} - e^{-c\tau_k}}{1-j\frac{\omega_k}{c}} \right) - \left(\frac{e^{c\tau_k} - e^{-c\tau_k}}{2} \right) e^{-(c+j\omega_k)T} \right] \\
&\quad + \sum_k \frac{b_{k,-1}}{2(1+j\frac{\omega_k}{c})} (e^{-c\tau_k} - e^{c\tau_k} e^{-2cT}) (1 - e^{-(c+j\omega_k)T}) \\
&\quad \left. + \sum_k \frac{b_{k,0}}{1+j\frac{\omega_k}{c}} \left[\frac{1}{1-j\frac{\omega_k}{c}} \left[e^{-c\tau_k} - e^{-j\omega_k \tau_k} e^{-(c-j\omega_k)T} \right] - \frac{1}{2} (e^{-c\tau_k} - e^{c\tau_k} e^{-2cT}) \right] \right\} \tag{5-23}
\end{aligned}$$

VI. EXACT ANALYSIS - SPECIAL CASE

Now, we put two constraints into all terms of exact analysis to form a special case :

$$1. \omega_k = \frac{2\pi kI}{T} \quad I: \text{positive integer.}$$

$$2. \tau_k = \tau \text{ for all adjacent channel } k.$$

For the purpose of normalization for future use, we set $\alpha = \frac{\tau}{T}$, or $\tau_k = \tau = \alpha I$ so that $\omega_k \tau_k = \omega_k \tau = \frac{2\pi kI}{T} \alpha T = 2\pi kI \alpha$

Still, we follow Table 5-2.

$$A. \int_0^T |S_B(t)|^2 dt = PT b_{0.0}^2 \left[1 - \frac{2(1-e^{-cT})}{cT} + \frac{(1-e^{-2cT})}{2cT} \right] \quad (6-1)$$

$$B. \int_0^T |S_{ISI}(t)|^2 dt = \frac{PT}{2cT} (1 - e^{-2cT}) \left[\sum_{i=-L_0}^{-1} b_{0,i} (e^{(i+1)cT} - e^{icT}) \right]^2 \quad (6-2)$$

C. Special case for ACI

C-1

$$\begin{aligned} \int_0^T |Z_0^T|^2 dt &= \frac{PT}{2cT} (1 - e^{-2cT}) \left| \sum_k \sum_{l=-L}^{-2} b_{k,l} \frac{e^{-j2\pi kl\alpha}}{1+j\frac{2\pi kl}{cT}} (e^{cT} - 1) e^{lcT} e^{\alpha cT} e^{j2\pi kl\alpha} \right|^2 \\ &= \frac{cTPT}{2} (e^{2\alpha cT} - e^{2(\alpha-1)cT}) (e^{cT} - 1)^2 + \sum_k \sum_{l=-2}^{-2} \frac{b_{k,l} e^{lcT}}{cT + j2\pi kl} \end{aligned} \quad (6-3)$$

C-2

$$\int_0^T |Z_0^\tau|^2 dt = \int_0^\tau |Z_0^\tau|^2 dt :$$

(a) $k \neq m, \omega_k \neq \omega_m$

$$\begin{aligned}
 &= (cT)^2 PT \sum_k \sum_m \frac{b_{k,-1} b_{m,-1}^*}{(cT+j2\pi kI)(cT-j2\pi mI)} \\
 &\quad \left[\frac{1-e^{-j2\pi I\alpha(k-m)}}{j2\pi I(k-m)} \right. \\
 &\quad + \frac{e^{-cT}-e^{(\alpha-1)cT+j2\pi mI\alpha}}{cT+j2\pi mI} \\
 &\quad + \frac{e^{-cT}-e^{(\alpha-1)cT-j2\pi kI\alpha}}{cT-j2\pi kI} \\
 &\quad \left. + \frac{e^{2(\alpha-1)cT}-e^{-2cT}}{2cT} \right] \tag{6-4a}
 \end{aligned}$$

(b) $k = m, \omega_k = \omega_m$

$$\begin{aligned}
 &= (cT)^2 PT \sum_k \frac{|b_{k,-1}|^2}{(cT)^2+(2\pi kI)^2} \left[\alpha + \frac{(e^{2(\alpha-1)cT}-e^{-2cT})}{2cT} \right. \\
 &\quad \left. + 2Re\left\{\frac{e^{-cT}-e^{(\alpha-1)cT+j2\pi kI\alpha}}{cT+j2\pi kI}\right\} \right] \tag{6-4b}
 \end{aligned}$$

C-3 3 terms for $|Z_\tau^T|^2$

C-3-1 $\int_0^T |Z_{\tau,-1}^T|^2 dt = \int_\tau^T |Z_{\tau,-1}^T|^2 dt$

(a) $k \neq m, \omega_k \neq \omega_m$

$$= \frac{1}{2} cT PT (1 - e^{-cT})^2 (1 - e^{2(\alpha-1)cT}) \sum_k \sum_m \frac{b_{k,-1} b_{m,-1}^*}{(cT + j2\pi kI)(cT - j2\pi mI)} \quad (6-5a)$$

(b) $k = m, \omega_k = \omega_m$

$$= \frac{1}{2} cT PT (1 - e^{-cT})^2 (1 - e^{2(\alpha-1)cT}) \sum_k \frac{|b_{k,-1}|^2}{(cT)^2 + (2\pi kI)^2} \quad (6-5b)$$

C-3-2 $\int_0^T |Z_{\tau,0}^T|^2 dt = \int_\tau^T |Z_{\tau,0}^T|^2 dt$

(a) $k \neq m, \omega_k \neq \omega_m$

$$= (cT)^2 PT \sum_k \sum_m \frac{b_{k,0} b_{m,0}^*}{(cT + j2\pi kI)(cT - j2\pi mI)}$$

$$\left[\frac{(e^{-j2\pi l\alpha(k-m)} - 1)}{j2\pi l(k-m)} \right]$$

$$+ \frac{(e^{(\alpha-1)cT + j2\pi m l \alpha} - 1)}{cT + j2\pi m l}$$

$$+ \frac{(e^{(\alpha-1)cT - j2\pi k l \alpha} - 1)}{cT - j2\pi k l}$$

$$+ \frac{(1 - e^{2(\alpha-1)cT})}{2cT} \quad (6-6a)$$

$$(b) \quad k = m, \quad \omega_k = \omega_m$$

$$\begin{aligned}
&= (cT)^2 PT \sum_k \frac{|b_{k,0}|^2}{(cT)^2 + (2\pi kI)^2} \left[1 - \alpha + \frac{(1-e^{2(\alpha-1)cT})}{2cT} \right. \\
&\quad \left. + 2Re \left\{ \frac{e^{(\alpha-1)cT+j2\pi kI\alpha}-1}{cT+j2\pi kI} \right\} \right] \tag{6-6b}
\end{aligned}$$

C-3-3

$$\int_0^T 2Re \left\{ Z_{\tau,-1}^T Z_{\tau,0}^{T^*} \right\} dt = \int_{\tau}^T 2Re \left\{ Z_{\tau,-1}^T Z_{\tau,0}^{T^*} \right\} dt :$$

$$(a) \quad k \neq m, \quad \omega_k \neq \omega_m$$

$$\begin{aligned}
&= 2(cT)^2 PT Re \left\{ \sum_k \sum_m \frac{b_{k,-1} b_{m,0}^* (1-e^{-cT})}{(cT+j2\pi kI)(cT-j2\pi mI)} \cdot \right. \\
&\quad \left. \left[\frac{(1-e^{(\alpha-1)cT+j2\pi mI\alpha})}{cT+j2\pi mI} + \frac{e^{2(\alpha-1)cT}-1}{2cT} \right] \right\} \tag{6-7a}
\end{aligned}$$

$$(b) \quad k = m, \quad \omega_k = \omega_m$$

$$\begin{aligned}
&= 2(cT)^2 PT Re \left\{ \sum_k \frac{b_{k,-1} b_{k,0}^* (1-e^{-cT})}{(cT)^2 + (2\pi kI)^2} \cdot \right. \\
&\quad \left. \left[\frac{(1-e^{(\alpha-1)cT+j2\pi kI\alpha})}{cT+j2\pi kI} + \frac{e^{2(\alpha-1)cT}-1}{2cT} \right] \right\} \tag{6-7b}
\end{aligned}$$

C-4

$$\int_0^T 2Re \{ Z_0^T Z_0^{T*} \} dt = \int_0^{\tau} 2Re \{ Z_0^T Z_0^{T*} \} dt$$

(a) $k \neq m, \omega_k \neq \omega_m$

$$= 2(cT)^2 PT Re \left\{ \sum_k \sum_m \sum_l^{-2} \frac{b_{k,l} b_{m,-1}^* (e^{(l+1)cT} - e^{lcT})}{(cT + j2\pi kI)(cT - j2\pi mI)} \cdot \right.$$

$$\left. \left[\frac{e^{\alpha cT + j2\pi mI\alpha} - 1}{cT + j2\pi mI} + \frac{e^{-cT} - e^{(2\alpha-1)cT}}{2cT} \right] \right\} \quad (6-8a)$$

(b) $k = m, \omega_k = \omega_m$

$$= 2(cT)^2 PT Re \left\{ \sum_k \sum_l^{-2} \frac{b_{k,l} b_{k,-1}^* (e^{(l+1)cT} - e^{lcT})}{(cT)^2 + (2\pi kI)^2} \right.$$

$$\left. \left[\frac{e^{\alpha cT + j2\pi kI\alpha} - 1}{cT + j2\pi kI} + \frac{e^{-cT} - e^{(2\alpha-1)cT}}{2cT} \right] \right\} \quad (6-8b)$$

C-5

$$\int_0^T 2Re \{ Z_0^T Z_{\tau}^{T*} \} dt = \int_{\tau}^T 2Re \{ Z_0^T Z_{\tau}^{T*} \} dt$$

$$= 2(cT)^2 PT Re \left\{ \sum_k \sum_m \sum_l^{-2} \frac{e^{(l+1)cT} - e^{lcT}}{(cT + j2\pi kI)(cT - j2\pi mI)} \left[\frac{b_{k,l} b_{m,-1}^* (1 - e^{2(\alpha-1)cT})(1 - e^{-cT})}{2cT} \right. \right.$$

$$\left. \left. + \frac{b_{k,l} b_{m,0}^* (1 - e^{(\alpha-1)cT + j2\pi mI\alpha})}{cT + j2\pi mI} + \frac{b_{k,l} b_{m,0}^* (e^{2(\alpha-1)cT} - 1)}{2cT} \right] \right\} \quad (6-9)$$

C-6

$$\int_0^T 2Re \{ Z_0^{\tau} Z_{\tau}^{T*} \} dt = \int_{\tau}^T 2Re \{ Z_0^{\tau} Z_{\tau}^{T*} \} dt = 0 \quad (6-10)$$

$$\begin{aligned}
D. \quad & \int_0^T 2Re \{ S_B(t) S_{ISI}^*(t) \} dt \\
& = \frac{PT b_{0,0}}{cT} (1 - e^{-cT})^2 \cdot \sum_{i=-L_0}^{-1} b_{0,i} (e^{(i+1)cT} - e^{icT}) \quad (6-11)
\end{aligned}$$

$$\begin{aligned}
E. \quad & \int_0^T 2Re \{ S_B^*(t) S_{ACI}(t) \} dt \\
& = 2PT Re \left\{ \sum_k \sum_l \frac{b_{0,0} b_{k,l} (1 - e^{-cT})^2 (e^{(\alpha+1)cT} - e^{\alpha cT}) e^{lcT}}{2(cT + j2\pi k l)} \right. \\
& \quad + \sum_k \frac{b_{0,0} b_{k,-1} cT}{cT + j2\pi k l} \left[\frac{(1 - e^{-j2\pi k l \alpha})}{j2\pi k l} + \frac{(e^{-\alpha cT} - e^{-j2\pi k l \alpha})}{cT - j2\pi k l} - \frac{(1 - e^{-\alpha cT})^2 e^{(\alpha-1)cT}}{2cT} \right] \\
& \quad + \sum_k \frac{b_{0,0} b_{k,-1} (2 - 2e^{(\alpha-1)cT} - e^{-\alpha cT} + e^{(\alpha-2)cT}) (1 - e^{-cT})}{2(cT + j2\pi k l)} \\
& \quad \left. + \sum_k \frac{b_{0,0} b_{k,0} cT}{cT + j2\pi k l} \left[\frac{(e^{-j2\pi k l \alpha} - 1)}{j2\pi k l} + \frac{(e^{-cT} - j2\pi k l \alpha - e^{-\alpha cT})}{cT - j2\pi k l} - \frac{(2 - 2e^{(\alpha-1)cT} - e^{-\alpha cT} + e^{(\alpha-2)cT})}{2cT} \right] \right\} \quad (6-12)
\end{aligned}$$

$$\begin{aligned}
F. \quad & \int_0^T 2Re \{ S_{ISI}^*(t) S_{ACI}(t) \} dt \\
& = 2PT \left[\sum_{i=-L_0}^{-1} b_{0,i} (e^{(i+1)cT} - e^{icT}) \right] \\
& \quad Re \left\{ \sum_k \sum_l \frac{b_{k,l} (1 - e^{-2cT}) (e^{(\alpha+1)cT} - e^{\alpha cT}) e^{lcT}}{2(cT + j2\pi k l)} \right. \\
& \quad + \sum_k \frac{b_{k,-1} cT}{cT + j2\pi k l} \left[\frac{(e^{-j2\pi k l \alpha} - e^{-\alpha cT})}{cT - j2\pi k l} - \frac{(e^{(\alpha-1)cT} - e^{-(\alpha+1)cT})}{2cT} \right] \\
& \quad + \sum_k \frac{b_{k,-1} (e^{-\alpha cT} - e^{(\alpha-2)cT}) (1 - e^{-cT})}{2(cT + j2\pi k l)} \\
& \quad \left. + \sum_k \frac{b_{k,0} cT}{cT + j2\pi k l} \left[\frac{(e^{-\alpha cT} - e^{-cT} - j2\pi k l \alpha)}{cT - j2\pi k l} - \frac{(e^{-\alpha cT} - e^{(\alpha-2)cT})}{2cT} \right] \right\} \quad (6-13)
\end{aligned}$$

APPENDIX B

Programs

% exact analysis with optimal threshold signal out of the integrator, the signal X contains desired signal and ACI & ISI and postdetect noise ,The formula for the BER is :

$$\frac{1}{2^{M(L+1)+L_0}} \sum_{2^M pattern} p(b)$$

$$where \ p(b) = \frac{1}{2} \left(\frac{1}{2\pi} \right)^M \left(\frac{1}{T} \right)^M$$

$$\left[\int_{-M/2}^{2\pi} \int_{-M/2}^T \dots \int_{-M/2}^T Q\left(\frac{\alpha - X_0(\phi_{-M/2} \dots \phi_{M/2} \tau_{-M/2} \dots \tau_{M/2})}{\sqrt{N_0 T}}\right) d\phi_{-M/2} \dots d\phi_{M/2} d\tau_{-M/2} \dots d\tau_{M/2} \right. \\ \left. + \int_{-M/2}^{2\pi} \int_{-M/2}^T \dots \int_{-M/2}^T Q\left(\frac{X_1(\phi_{-M/2} \dots \phi_{M/2} \tau_{-M/2} \dots \tau_{M/2}) - \alpha}{\sqrt{N_0 T}}\right) d\phi_{-M/2} \dots d\phi_{M/2} d\tau_{-M/2} \dots d\tau_{M/2} \right]$$

M is the number of adjacent channels. Here we assume all channels have the same phase ϕ and time delay τ i.e, $\phi_k = \phi$, $\tau_k = \tau$ hence for our model the power of $(1/2\pi)$ and $(1/T)$ are fixed to 1, therefore we only have two integrals and two arguments.

```

M=4; k=[-M/2:-1 1:M/2];
m=k;
% produce the controlled matrix b to control 64 different bit patterns
m1=[ zeros(1,32) ones(1,32) ];
m2=[ zeros(1,16) ones(1,16) zeros(1,16) ones(1,16) ];
m3=[];
for i=1:4
    m3=[m3 [zeros(1,8) ones(1,8)]]; 
end
m4=[];
for i=1:8
    m4=[m4 [zeros(1,4) ones(1,4)]]; 
end
m5=[];
for i=1:16
    m5=[m5 [zeros(1,2) ones(1,2)]]; 
end
m6=[];
for i=1:32
    m6=[m6 zeros(1,1) ones(1,1)]; 
end

```

```

b=[m1;m2;m3;m4;m5;m6]';

% signal to noise ratio range in dB
RPTN_DB=[10:0.5:25]; %RPTN_dB
ppp=10.^0.1*RPTN_DB;
len1=length(ppp);

% solalph function is provided by Professor Randy L. Borchardt

n=10;
[bp, wfx]=grule(n); %bp=bp, wfx=wfx

% single channel
BER0=0.5*erfc(ppp/8^0.5);

pp=[]; thresh=[]; %thresh is not normalized threshold

for CT=[5 10 15 20] % cT is Fabry-Perot filter parameter
    if CT==5
        qqq=[2:12 17 20];
    elseif CT==10
        qqq=[3:12 17 20];
    elseif CT==15
        qqq=[4:12 17 20];
    elseif CT==20
        qqq=[5:12 17 20];
    end

pe=[]; thresh1=[];

for I=qqq
    BER=[]; thresh2=[];

    for RPTN=ppp
        ap=linspace(0,182,11); % approximated thresholds
        x3=0; % first few loops to find out the threshold which make the
        % BER minimum don't care about the scale
        for i=1:64 x3=x3+...
            solalph('xx00',0,2*pi,2,bp, wfx,0,1,2,bp, wfx,ap,b,CT,i,I,k,m,RPTN)+...
            solalph('xx11',0,2*pi,2,bp, wfx,0,1,2,bp, wfx,ap,b,CT,i,I,k,m,RPTN);
        end
        [val,ind]=min(x3);
        lef=ap(ind)-16.6;
    end
end

```

```

if lef<0 ,lef=0; end      % to aviod the threshold go beyond the negative side

ap=linspace(lef,ap(ind)+16.6,11);
% two more time to find alpha
x3=0;

for i=1:64  x3=x3+...
    solalph('xx00',0,2*pi,2,bpx,wfx,0,1,2,bpx,wfx,ap,b,CT,i,I,k,m,RPTN)+...
    solalph('xx11',0,2*pi,2,bpx,wfx,0,1,2,bpx,wfx,ap,b,CT,i,I,k,m,RPTN);
end

[val,ind]=min(x3);
lef=ap(ind)-3.1;

if lef<0 ,lef=0; end

ap=linspace(lef,ap(ind)+3.1,11);
% three more time to find alpha
x3=0;

for i=1:64
    x3=x3+...
    solalph('xx00',0,2*pi,2,bpx,wfx,0,1,2,bpx,wfx,ap,b,CT,i,I,k,m,RPTN)+...
    solalph('xx11',0,2*pi,2,bpx,wfx,0,1,2,bpx,wfx,ap,b,CT,i,I,k,m,RPTN);
end

[val,ind]=min(x3);
lef=ap(ind)-0.57;

if lef<0 ,lef=0; end

ap=linspace(lef,ap(ind)+0.57,8);
% four more time to find alpha
x3=0;

for i=1:64
    x3=x3+...
    solalph('xx00',0,2*pi,2,bpx,wfx,0,1,2,bpx,wfx,ap,b,CT,i,I,k,m,RPTN)+...
    solalph('xx11',0,2*pi,2,bpx,wfx,0,1,2,bpx,wfx,ap,b,CT,i,I,k,m,RPTN);
end

[val,ind]=min(x3);
ap=ap(ind);

```

```

% after find optimal alpha use double integration
qq3=0;

for i=1:64
    qq3=qq3+( dbgquadm('xx00',0,2*pi,2,bpx,wfx,0,1,2, ...
        bpx,wfx,ap,b,CT,i,I,k,m,RPTN)+...
        dbgquadm('xx11',0,2*pi,2,bpx,wfx,0,1,2, ...
        bpx,wfx,ap,b,CT,i,I,k,m,RPTN) )/(256*pi);
end

BER=[BER qq3]; thresh2=[thresh2 ap];

if qq3<10^(-15)      %for save time only interesting in 10^(-15)
    ii=find(ppp==RPTN);
    BER(ii+1:len1)=5*10^(-116)*ones( 1,length(ii+1:len1) );
    thresh2(ii+1:len1)=(ap+2)*ones(1,length(ii+1:len1) );
    break
end
end

pe=[pe;BER]; thresh1=[thresh1;thresh2];
end

pp=[pp;pe]; thresh=[thresh;thresh1];
end

time2=toc;

```

```

% worse case form appendix setting optimal threshold equal (x0+x1)/2
M=4;
I=linspace(0,6,101); % set I value in linspace to find minimum I
RPTN_DB=0:0.2:20; % x-axis dB range
RPTN=10.^0.1*RPTN_DB); % change to ratio
% single channel
BER0=0.5*erfc(RPTN/8^0.5);
%find optimal alpha
pp=[];
for CT=[5 10 15 20]
    x3=0;
    for k=1:0.5*M
        x3=x3+2./(1+(2*pi*k*I/CT).^2);
    end
    x0=(1-exp(-2*CT))/(2*CT)+x3*(1+2*(1-exp(-CT))/CT);
    x1=1-2*(1-exp(-CT))/CT+(1-exp(-2*CT))/(2*CT);
    [val,ind]=min(abs(x0-x1));
    I=ceil(I(ind));
    I=(I(ind)); % our minimum I value

% set different I and to find the BER
ss=I+1:I+10;
BER=[];
for I=ss
    x3=0;
    for k=1:0.5*M
        x3=x3+2/(1+(2*pi*k*I/CT)^2);
    end
    x0=(1-exp(-2*CT))/(2*CT)+x3*(1+2*(1-exp(-CT))/CT);
    x1=1-2*(1-exp(-CT))/CT+(1-exp(-2*CT))/(2*CT);
    BER=[BER;0.5*erfc( RPTN*(x1-x0)/8^0.5 ) ];
end
pp=[pp;BER];
end

semilogy(RPTN_DB,BER0,'--',RPTN_DB,pp(:,1))

axis([10 19 10^(-15) 1])

```

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